

13. Problems from the Textbook: 1–41, 45, 53, 65, 67 (2.2); 1, 3, 5, 7, 8, 9, 11, 13, 19, 23 (2.3); 1, 3, 5, 7, 9, 11, 13, 15, 21, 23, 25, 27, 29, 31, 33, 35, 37, 45, 47, 51 (2.4); 1–29, 33, 35, 39, 43, 53, 57, 59, 61 (2.5); 2, 3, 5, 7, 9, 13, 17, 21, 23, 27, 31 (2.6); 3, 5, 7, 9, 19, 23, 25, 27, 29, 39 (2.7).
14. Find the derivatives of the following functions by using the definition of the derivative.
- (a) $f(x) = \frac{1}{\sqrt{x}}$ (b) $g(x) = 3x^3 + 2x^2 - 5x + 6$
 (c) $h(x) = \frac{2-3x}{3-2x}$ (d) $p(x) = \sqrt{x^2 + 4}$.
15. Find a and b such that $H(x) = \begin{cases} ax & \text{for } x \leq 1 \\ 2x^2 + b & \text{for } x > 1 \end{cases}$ is continuous and differentiable on \mathbb{R} .
16. Let $f(x) = x^2$.
- (a) Find the equation of the tangent of f at 3.
 (b) Find the equation of the normal line of f at 3.
 (c) Where is the tangent of f at 5 crossing the x -axis?
 (d) Find all points x_0 such that the tangent of f at x_0 passes through the origin.
 (e) Find all points x_0 such that the tangent of f at x_0 passes through $(-5, 6)$.
17. Find numbers a, b, c such that $f(x) = ax^2 + bx + c$ has the zeros 0 and 5 and a tangent at 2 with slope 1.
18. Find the derivatives of the following functions.
- (a) $f(t) = t^4 + 5t^2 - 8t + 100$ (b) $f(t) = \frac{1}{t^2} + \frac{1}{t^3}$
 (c) $h(x) = \frac{x^2 - 2x + 1}{x - 2}$ (d) $f(x) = (3x - 1)^8$
 (e) $f(x) = \sqrt{x}(x - 3)^8$ (f) $f(x) = (x^3 - 1)^8(3x^2 + 5x)^7$
 (g) $g(x) = \frac{(x-1)^5}{(x+2)^4}$ (h) $h(x) = \sqrt{x^2 + 5}$
 (i) $f(x) = \sqrt{\frac{x^2 + 1}{x^2 - 5}}$ (j) $\alpha(x) = \sqrt{x^2 + \sqrt{x}}$.
19. Let $f(x) = x^4$.
- (a) Find $f'(x), f''(x), f'''(x), f^{(4)}(x),$ and $f^{(5)}(x)$.
 (b) Find $f'(0)x + f''(0)\frac{x^2}{2} + f'''(0)\frac{x^3}{6} + f^{(4)}(0)\frac{x^4}{24}$.
 (c) Find $f'(1)(x - 1) + f''(1)\frac{(x-1)^2}{2} + f'''(1)\frac{(x-1)^3}{6} + f^{(4)}(1)\frac{(x-1)^4}{24}$.
 (d) Find $f'(2)(x - 2) + f''(2)\frac{(x-2)^2}{2} + f'''(2)\frac{(x-2)^3}{6} + f^{(4)}(2)\frac{(x-2)^4}{24}$.
 (e) Find $f'(3)(x - 3) + f''(3)\frac{(x-3)^2}{2} + f'''(3)\frac{(x-3)^3}{6} + f^{(4)}(3)\frac{(x-3)^4}{24}$.
 (f) Try to generalize the above results.
20. A function f^{-1} is called the inverse of f if $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$ for all x .
- (a) Let $f(x) = 3x + 1$. Find $f^{-1}(x)$ and $(f^{-1})'(x)$.
 (b) Let $g(x) = \frac{1}{4x+5}$. Find $g^{-1}(x)$ and $(g^{-1})'(x)$.
 (c) Give a formula for $(f^{-1})'(x)$ when f^{-1} is the inverse of f .
 (d) If E is a function with $E'(x) = E(x)$, what is $(E^{-1})'(x)$?