- 1. Read Chapters 1 and 2 of the textbook.
- 2. For Example 1.1 from class, give four sets of amounts x_1, x_2, \ldots, x_8 that are not feasible (different from the ones presented in class) and give four sets of amounts x_1, x_2, \ldots, x_8 that are feasible (different from the ones presented in class), including the corresponding costs.
- 3. An oil concern produces three different kinds of crude oil, which are used to produce regular and premium gasoline. The amount of sulfur contained in each of the three kinds, the daily producing capacity of each of the three kinds, and the costs for producing each of the three kinds are given in the table below. A barrel of regular gasoline is allowed to contain at most 200 g sulfur, and its sales price is \$22. A barrel of premium gasoline can't contain more than 100 g sulfur, and its sales price is \$26. The problem is how much regular and how much premium gasoline should be produced in order to maximize the daily profit of the concern. Formulate a linear program that solves this problem and give four nonfeasible sets of amounts of production quantities, as well as four feasible sets including their corresponding daily profits.

| Crude oil | sulfur contained | capacity | $\cos t$ |
|-----------|------------------|-------------|-----------|
| | g/barrel | barrels/day | \$/barrel |
| 1 | 300 | 5000 | 6 |
| 2 | 100 | 3000 | 8 |
| 3 | 50 | 4000 | 12. |

4. Let
$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$
, $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$. Compute $2A$, B^T , $B + B^T$, AC , CB , ACB , A^2 , B^2 , CC^T .

- 5. Prove that matrix multiplication (with matrix addition) is distributive.
- 6. Only three brands of beer (beer 1, 2, and 3) are available for sale in Melbornolis. From time to time, people try one or another of these brands. Suppose that at the end of each month, people change the beer they are drinking according to the following rules: 30% of the people who drink beer 1 switch to beer 2, 40% switch to beer 3; 20% of beer 2 drinkers switch to beer 1, 50% to beer 3; 50% of beer 3 drinkers switch to beer 1, 5% to beer 2. Let x be the vector consisting of x_i (number of people drinking beer i at the beginning of the first month) and y be the vector consisting of y_i (number of people drinking beer i at the beginning of the second month). Relate the vectors x and y using matrices. If z is the vector consisting of z_i (number of people drinking beer i at the beginning of the third month), relate y and z. Also relate x and z.
- 7. For the following systems of equations, rewrite the systems as an equation Ax = b, do Gaussian Elimination and find the solution:
 - (a) 2u + 4v = 3, 3u + 7v = 2;
 - (b) 3u + 5v + 3w = 25, 7u + 9v + 19w = 65, -4u + 5v + 11w = 5;
 - (c) u + 2v + 3w = 39, u + 3v + 2w = 34, 3u + 2v + w = 26;
 - (d) u + 3v + 5w = 1, 3u + 12v + 18w = 1, 5u + 18v + 30w = 1.