Chapter 2

Probability

2.1 Sample Spaces and Events

2.2 Axioms, Interpretations, and Properties of Probability

2.3 Counting Techniques

2.4 Conditional Probability

2.5 Independence

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Experiments

- An **experiment** is any action or process whose outcome is subject to uncertainty.
- The **sample space** of an experiment is the set of all possible outcomes of that experiment.
- An **event** is any subset of outcomes contained in the sample space.
- An event is **simple** if it consists of exactly one outcome and **compound** otherwise.

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Example

Consider the following experiment: Roll a die.

Find the sample space of this experiment and specify/classify the following events:

a) It's a "3" (event A)

b) It's a "4" or a "5" (event B)

c) It's an even number (event C)

d) It's at least "4" (event D)

e) It's between "2" and "4", each included (event E)

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Example

Consider the following experiment: Pick a random number between 0 and 1.

Find the sample space of this experiment and specify/classify the following events:

a) It's less than a half (event A)

b) It's less than or equal to three quarters (event B)

c) It's a half (event C)

d) It's between one quarter and three quarters, each included (event D)

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Relations from Set Theory

- The **complement** of an event is the set of all outcomes in the sample space that are not contained in that event.
- The **union** of two events is the set of all outcomes that are either in one of these two sets or in both.
- The **intersection** of two events is the set of all outcomes that are in both of the events.
- Two events with empty intersection are called **mutually exclusive** or **disjoint**.

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Venn Diagrams

(a) Venn diagram of events A and B
(b) Shaded region is $A \cap B$
(c) Shaded region is $A \cup B$
(d) Shaded region is A'
(e) Mutually exclusive events

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Example

For the discrete (die) example, find the following events:

A', B', C', A∪B, A∪C, B∪C, A∪(B∪C), B∩C, B∩D, C∩D, A∩B, C∩(B∩D), (C∩D)∩B, A∪(B∩C), (A∪B)∩(A∪C), D∩(B∪C), (D∩B)∪(D∩C), (B∪C)', B'∩C', (B∩C)', B'∪C'.

For the continuous (random number) example, find the following events:

A∪B, A∪C, A', C', A∩B, A∩D, (A∩D)', D', A'∪D'.

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