Chapter 4

Continuous Random Variables

4.1 Probability Density Functions

4.2 Cumulative Distribution Functions and Expected Values

4.3 The Normal Distribution

4.4 The Exponential and Gamma Distributions

4.5 Other Continuous Distributions

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pdf

The probability distribution or probability density function (pdf) of a continuous rv X is a function f such that for any two numbers a and b with a≤b, we have

$$P(a \le X \le b) = \int_a^b f(x)dx$$

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Properties of the pdf

For f to be a legitimate pdf, it must satisfy the following two conditions:

$$f(x) \ge 0 \qquad \int_{-\infty}^{\infty} f(x)dx = 1$$

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Example

Define a function f by f(x)=0.5+cx if 0≤x≤1 and f(x)=0 otherwise. Suppose that f is the pdf of a continuous rv X.

- **Find the value of c that ensures that f is a pdf.**
- **What is the probability that X is less than 0.5?**
- **What is the probability that X is bigger than 0.5?**
- **What is the probability that X is equal to 0.5?**
- **What is the probability that X is between 3/4 and 5/6?**

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Example

Describe the pdf for the rv X that measures the angle of the final ball position (measured at the ball's center) along a roulette wheel's center.

- **What is the probability that X is between 0 and π?**
- **What is the probability that X is between π/2 and 3π/2?**
- **What is the probability that X is less than π/2?**

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Uniform Continuous RV

The uniform continuous rv X is the rv with the pdf

$$f(x;a,b) = \begin{cases} \dfrac{1}{b-a} & \text{for } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

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Example

Suppose the pdf of a continuous rv X is given by $f(x)=0.15\exp(-0.15(x-0.5))$ if $x \geq 0.5$ and $f(x)=0$ otherwise.

- **Verify that f is a pdf.**
- **What is the probability that X is at most 5?**
- **What is the probability that X is at least 10?**