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Chapter 4

Continuous Random Variables

- 4.1 Probability Density Functions
- 4.2 Cumulative Distribution Functions and Expected Values**
- 4.3 The Normal Distribution
- 4.4 The Exponential and Gamma Distributions
- 4.5 Other Continuous Distributions

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cdf

The cumulative distribution function (cdf) of a continuous rv X is defined for every number x by

$$F(x) = P(X \leq x)$$

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Example

Define a function f by $f(x) = 0.5 + x$ if $0 \leq x \leq 1$ and $f(x) = 0$ otherwise. Suppose that f is the pdf of a continuous rv X.

- Find the cdf of X.
- What is the probability that X is less than 0.5?
- What is the probability that X is bigger than 0.5?
- What is the probability that X is between 3/4 and 5/6?

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Example

Find the cdf of a uniform continuous rv.

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Example

Suppose the pdf of a continuous rv X is given by $f(x) = 0.15 \exp(-0.15(x-0.5))$ if $x \geq 0.5$ and $f(x) = 0$ otherwise.

- Find the cdf of X.
- What is the probability that X is at most 5?
- What is the probability that X is at least 10?

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cdf - pdf

The pdf f of a continuous rv X can be obtained from the cdf F when F' exists:

$$f(x) = F'(x)$$

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Example

Suppose the cdf of a continuous rv X is given by $F(x)=0$ for $x<0$, $F(x)=x/8+3x^2/16$ for $0\leq x\leq 2$, and $F(x)=1$ for $x>2$.
Find the pdf of X .

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Percentile and Median

Let $0\leq p\leq 1$. The **(100) p th percentile $\eta(p)$** of a continuous rv X is defined by
 $F(\eta(p))=p$.
 The **median** of X is the 50th percentile.

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Example

Suppose the cdf of a continuous rv X is given by $F(x)=0$ for $x<0$, $F(x)=x/8+3x^2/16$ for $0\leq x\leq 2$, and $F(x)=1$ for $x>2$.
 - Find the 20th percentile of X .
 - Find the 70th percentile of X .
 - Find the median of X .

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Expected Value

Let X be a continuous rv with pdf f . The **expected value or mean value** of X is defined by

$$E(X) = \mu_X = \mu = \int_{-\infty}^{\infty} x \cdot f(x) dx$$

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Example

Find the expected value of the continuous uniform random variable.

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Example

Suppose the pdf of a continuous rv X is given by $f(x)=0.15\exp(-0.15(x-0.5))$ if $x\geq 0.5$ and $f(x)=0$ otherwise.
Find $E(X)$.

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Expected Value of a Function

Let X be a continuous rv with pdf f . The **expected value** of a function $h(X)$ is

$$E(h(X)) = \mu_{h(X)} = \int_{-\infty}^{\infty} h(x) \cdot f(x) dx$$

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Properties of Expectation

1. $E(aX) = aE(X)$ for all numbers a
2. $E(X+b) = E(X) + b$ for all numbers b
3. $E(aX+b) = aE(X) + b$ for all numbers a, b

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Variance

Let X be a continuous rv with pdf f . The **variance** of X is defined by

$$V(X) = E((X - E(X))^2) = \sigma_X^2 = \int_{-\infty}^{\infty} (x - \mu)^2 \cdot f(x) dx$$

The root of the variance is called the **standard deviation (SD)** of X

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Example

Find the variance of the continuous uniform random variable on $[0, 1]$.

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A Shortcut Formula for the Variance

An alternative expression for the variance is

$$V(X) = E(X^2) - (E(X))^2$$

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Example

Define a function f by $f(x) = 0.5 + x$ if $0 \leq x \leq 1$ and $f(x) = 0$ otherwise. Suppose that f is the pdf of a continuous rv X .

- Find $E(X)$.
- Find $E(X^2)$.
- Find $V(X)$.

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Properties of the Variance

1. $V(aX+b)=a^2V(X)$ for all numbers a, b
2. $V(X+b)=V(X)$ for all numbers b
3. $V(aX)=a^2V(X)$ for all numbers a
4. $\sigma_{aX+b}=|a|\sigma_X$ for all numbers a, b
5. $\sigma_{aX}=|a|\sigma_X$ for all numbers a
6. $\sigma_{X+b}=\sigma_X$ for all numbers b

