Chapter 4

Continuous Random Variables

4.1 Probability Density Functions
4.2 Cumulative Distribution Functions and Expected Values
4.3 The Normal Distribution
4.4 The Exponential and Gamma Distributions
4.5 Other Continuous Distributions

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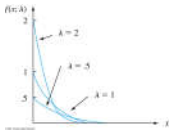
Exponential RV

The exponential rv X with parameter λ is the rv with the pdf (for x≥0)



$$f(x;\lambda) = \lambda e^{-\lambda x}$$

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Expectation and Variance of the exponential RV

If X is an exponential rv with parameter λ, then

$E(X)=1/\lambda$

$V(X)=1/\lambda^2$

$F(x;\lambda)=1-e^{-\lambda x}$ for $x\geq 0$

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Example

Assume that the rv describing the service life of an electronic device is exponentially distributed with mean 500 hours.

Find the probability that the service life of the device is between 300 and 600 hours.

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Example

Assume that the rv describing the time difference between arrival of two customers at a certain office is exponentially distributed. Suppose the probability that no customer arrives within at least two minutes is 0.1.

What is the probability that no customer arrives within at least four minutes?

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Memoryless Property

The exponential rv X with parameter λ has the memoryless property:

$P(X\leq x+h \mid X\geq x)=P(X\leq h)$

for all $x\geq 0$ and $h>0$

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Gamma Function

The gamma function

$$\Gamma(\alpha)=\int_0^\infty x^{\alpha-1}e^{-x}dx$$

has the following properties:

1. $\Gamma(\alpha)=(\alpha-1)\Gamma(\alpha-1)$ for any number $\alpha>1$
2. $\Gamma(n)=(n-1)!$ for any positive integer n
3. $\Gamma(1/2)=\pi^{1/2}$

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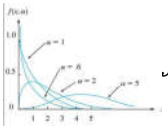
Standard Gamma RV

The standard gamma rv X with parameter $\alpha>0$ is the rv with the pdf (for $x\geq 0$)



$$f(x;\alpha)=\frac{x^{\alpha-1}e^{-x}}{\Gamma(\alpha)}$$

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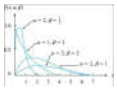
Gamma RV

The gamma rv X with parameters $\alpha>0$ and $\beta>0$ is the rv with the pdf (for $x\geq 0$)



$$f(x;\alpha,\beta)=\frac{x^{\alpha-1}e^{-x/\beta}}{\beta^{\alpha}\Gamma(\alpha)}$$

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Expectation and Variance of the Gamma RV

If X is a Gamma rv with parameters α and β, then

$E(X)=\alpha\beta$

$V(X)=\alpha\beta^2$

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cdf of the Gamma RV

the cdf $F(x;\alpha)$ of the standard gamma rv is called the incomplete Gamma function.

The cdf of the gamma rv is $F(x/\beta,\alpha)$

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Example

Suppose the reaction time X of a randomly selected individual to a certain stimulus is a standard gamma rv with $\alpha=2$.

What is the expectation and variance of X?

What is the probability that reaction time is

- between 3 and 5 seconds?
- more than 4 seconds?

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Example

Assume that the survival time X (in weeks) of a mouse exposed to a certain radiation is a gamma rv with α=8 and β=15.

What is the expectation and variance of X?

What is the probability that survival time is

- between 60 and 120 weeks?

- more than 30 weeks?

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Chi-Squared RV

The chi-squared rv X with parameter (positive integer) v is the gamma rv with α=v/2 and β=2, i.e., it has pdf (for x≥0)

$$f(x;v)=\frac{x^{v/2-1}e^{-x/2}}{2^{v/2}\Gamma(v/2)}$$

v is called the number of degrees of freedom (df)

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