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Agarwal, Ravi P.; **Bohner, Martin**; **Grace, Said R.**; **O'Regan, Donal**

Discrete oscillation theory. (English)

New York, NY: Hindawi Publishing Corporation. xiii, 961 p. $ 275.00 (2005). [ISBN 977-5945-19-4/hbk]

This is a very comprehensive book dealing with the oscillation theory of various difference equations. The qualitative theory of difference equations attracted considerable attention in last years and really a very large number of papers dealing with various aspects of this theory appeared. It is sufficient to enter Zbl databasis and to search for discrete/difference oscillation theory. The reviewed book summarizes in a consistent form both classical and recent results in the oscillation theory of various difference equations. It is written by mathematicians who achieved remarkable results in this area. The book consists of nine chapters. The first one, devoted to the oscillation theory of second order difference equations, is by far the largest one (180 pages). It is shown there that the classical Sturmian theory for differential equations extends, when modified appropriately, to difference equations. The relationship between oscillatory properties of second order equations, solvability of the associated Riccati equation and positivity of the corresponding discrete quadratic functional is derived and many (non)oscillation criteria based on this relationship are presented. Chapter 2 generalizes the results of Chapter 1 to symplectic difference systems which cover as a special case linear Hamiltonian difference systems and many other equations and systems. The main problem there is to define appropriately the concept of a generalized zero of matrix-valued sequences. This was done in the fundamental paper of *M. Bohner* published in [J. Math. Anal. Appl. 199, 804-826 (1996; Zbl 0855.39018)], and enabled to elaborate oscillation theory similar to the scalar case. Another generalization of the results of Chapter 1 is the contents of Chapter 3 which deals with the oscillatory properties of half-linear differential equations. Most of the results of this chapter are due to *P. Rehák*, his fundamental paper in the half-linear oscillation theory [Czech. Math. J. 51, 303-321 (2001; Zbl 0982.39004)].

Chapters 4 and 5 are devoted to the oscillation theory of nonlinear second order difference equations. In addition to numerous (non)oscillation criteria, various other topics are discussed there, like classification of nonoscillatory solutions, boundedness of solutions, etc. The next Chapter 6 deals with second order difference equations with deviating argument (both advanced and retarded). Oscillation criteria for linear difference equations via their characteristic equation are given and many criteria for oscillation and almost oscillation of linear damped and forced difference equations with deviating argument are offered. There is a long years discussion about relationship between difference equation with deviating argument and higher order difference equations in the "difference equations community". The methods used in this chapter are typical for second order difference equations and reflect similarities between differential and difference equations with deviating argument. Chapter 7 is concentrated on neutral second order difference equations. Numerous (non)oscillation criteria for linear and nonlinear equations with and without forcing term are established. Another central topic treated

in this chapter is a classification scheme for nonoscillatory solutions. Finally, oscillation criteria for neutral equations of mixed type with constant or periodic coefficients are offered. The next Chapter 8 is devoted to the stability and oscillation theory for differential equations with piecewise constant arguments. These equations represent a hybrid of continuous and discrete dynamic systems and therefore combine properties of both differential and difference equations. Attention is mainly focused to first and second order equations and systems. The last section of this chapter deals with the problem of oscillation of positive solutions of the logistic equation with quadratic nonlinearity around its positive equilibrium. The last chapter of the book discusses miscellaneous topics of the discrete oscillation theory. As an example, let us mention linearized oscillations for difference equations or oscillation of recursive sequences. The book is addressed to a wide audience of specialists. Not only mathematicians, but also physicists, engineers, and biologists will find there useful information. The book can be used as a textbook at the graduate level and as a reference book, the bibliography contains almost 300 items. It is written at a level easy to understand for college students who have had courses in calculus. By my opinion, every researcher in the qualitative theory of difference equations should have this book in his/her bookshelf.

Ondřej Došlý (Brno)

Keywords : Discrete oscillation theory; Sturm-Liouville diference equation; symplectic difference system; half-linear difference equation; nonlinear discrete oscillation; differential equations with piecewise constant arguments; textbook; Riccati equation; linear Hamiltonian difference systems; nonoscillatory solutions; boundedness of solutions; neutral second order difference equations; stability; discrete dynamic systems; positive equilibrium

Classification :

∗39A11 Stability of difference equations
34C10 Qualitative theory of oscillations of ODE: Zeros, etc.
39A10 Difference equations
39-02 Research monographs (functional equations)
39A12 Discrete version of topics in analysis

Cited in ...