

# Fixed Income Models

Winter 2010/2011 Lecture Notes

Martin Bohner

*Version from January 31, 2011*

DEPARTMENT OF MATHEMATICS AND STATISTICS, MISSOURI UNIVERSITY OF  
SCIENCE AND TECHNOLOGY, ROLLA, MISSOURI 65409-0020, USA

*Current address:* Universität Ulm, Department of Mathematical Finance,  
Helmholtzstrasse 18, D-89069 Ulm, Germany

*E-mail address:* `bohner@mst.edu`

*URL:* `http://web.mst.edu/~bohner`



# Contents

|                                                  |    |
|--------------------------------------------------|----|
| Chapter 1. Interest Rates                        | 1  |
| Chapter 2. Interest Rate Derivatives             | 5  |
| 2.1. Forward Rate Agreements                     | 5  |
| 2.2. Interest Rate Swaps                         | 5  |
| 2.3. Interest Rate Caps and Floors               | 7  |
| 2.4. Swaptions                                   | 9  |
| Chapter 3. Pricing Theory                        | 11 |
| 3.1. Preliminaries from Financial Mathematics    | 11 |
| 3.2. Three Examples of Numéraires                | 13 |
| 3.3. Pricing Formulas                            | 14 |
| 3.4. Two Useful Formulas                         | 16 |
| Chapter 4. One-Factor Short-Rate Models          | 17 |
| 4.1. Vasicek Model                               | 17 |
| 4.2. Exponential Vasicek Model                   | 20 |
| 4.3. Dothan Model                                | 21 |
| 4.4. Cox–Ingersoll–Ross Model                    | 21 |
| 4.5. Affine Term-Structure Models                | 23 |
| Chapter 5. Extended One-Factor Short-Rate Models | 25 |
| 5.1. Ho–Le Model                                 | 25 |
| 5.2. Hull–White Model (Extended Vasicek Model)   | 26 |
| 5.3. Black–Karasinski Model                      | 29 |
| 5.4. Deterministic-Shift Extended Models         | 30 |
| 5.5. Extended CIR Model                          | 31 |
| 5.6. Extended Affine Term-Structure Models       | 32 |

|                                                   |    |
|---------------------------------------------------|----|
| Chapter 6. Two-Factor Short-Rate Models           | 33 |
| 6.1. G2++ Model                                   | 33 |
| 6.2. Hull–White Two-Factor Model                  | 38 |
| 6.3. CIR2 Model                                   | 39 |
| 6.4. Longstaff–Schwartz Model                     | 40 |
| 6.5. CIR2++ Model                                 | 41 |
| Chapter 7. Heath–Jarrow–Morton Framework          | 43 |
| 7.1. Heath–Jarrow–Morton Model                    | 43 |
| 7.2. Gaussian HJM Model                           | 45 |
| 7.3. Ritchken–Sankarasubramanian Model            | 45 |
| 7.4. Mercurio–Moralada Model                      | 49 |
| Chapter 8. Market Models                          | 51 |
| 8.1. Lognormal Forward-LIBOR Model                | 51 |
| 8.2. Lognormal Forward-Swap Model                 | 52 |
| Chapter 9. The Volatility Smile                   | 53 |
| 9.1. The Smile Problem                            | 53 |
| 9.2. Shifted Lognormal Model                      | 53 |
| 9.3. Brigo–Mercurio Local Volatility Model        | 54 |
| 9.4. Lognormal Mixture Model                      | 55 |
| 9.5. Lognormal Mixture Model with Different Means | 56 |
| 9.6. Second Brigo–Mercurio Local Volatility Model | 57 |
| 9.7. Geometric Brownian Motion Mixture Model      | 59 |
| Chapter 10. Pricing of Further Derivatives        | 61 |
| 10.1. In-Arrears Swaps                            | 61 |
| 10.2. In-Arrears Caps                             | 62 |
| 10.3. Caps with Deferred Caplets                  | 63 |
| Bibliography                                      | 65 |