Winter Term 10/11

INSTITUTE OF MATHEMATICAL FINANCE

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Fixed Income Models

Exercise Sheet 1

(Due: Tuesday 11/02/2010)

1. **(Compounding Frequencies)**
You invest €100 for 1 year at 10% p.a. What is the value of your investment after 1 year when the interest is compounded
   - annually;
   - quarterly;
   - daily;
   - continuously?

2. **(Various Interest Rates)**
Consider the following table of German treasury bond prices as of 11/03/2009:

Maturity	Price	Time to Maturity (years)	Maturity	Price	Time to Maturity (years)
02/24/2010	99,82	0,31	03/31/2010	99,75	0,41
04/28/2010	99,60	0,48	05/19/2010	99,43	0,54
07/04/2010	99,16	0,67	01/04/2011	98,50	1,17
07/04/2011	97,54	1,67	01/04/2012	96,34	2,17
07/04/2012	94,94	2,67	01/04/2013	93,46	3,17
07/04/2013	92,16	3,67	01/04/2014	90,35	4,17
07/04/2014	88,97	4,67	01/04/2015	87,12	5,17
07/04/2015	85,52	5,67	01/04/2016	83,76	6,17
07/04/2016	81,82	6,67	01/04/2017	80,07	7,17
07/04/2017	78,79	7,67	01/04/2018	77,01	8,17
07/04/2018	75,34	8,67	01/04/2019	72,82	9,17
07/04/2019	70,00	9,67	01/04/2020	69,92	10,10
07/04/2020	69,20	10,67	01/04/2021	66,86	11,17
07/04/2021	64,57	11,67	01/04/2022	62,78	12,17
07/04/2022	60,77	12,67	01/04/2023	60,25	13,17
07/04/2023	57,51	13,67	01/04/2024	56,36	14,17
07/04/2024	54,61	14,67	01/04/2025	55,50	15,17
07/04/2025	51,62	15,67	01/04/2026	50,22	16,17
07/04/2026	51,50	16,67	01/04/2027	47,10	17,17
07/04/2027	46,00	17,67	01/04/2028	44,68	18,17
07/04/2028	44,02	18,67	01/04/2029	42,89	19,17
07/04/2029	44,00	19,67			

Use the market data to calculate the following (continuously compounded) interest rates:
   - The forward rate at 11/03/09 from 01/04/15 to 01/04/17;
   - The forward rate at 11/03/09 from 07/04/19 to 01/04/23;
   - The spot rate at 11/03/09 to 07/04/10;
   - The instantaneous forward rate at 11/03/09 for 04/28/10 approximately;
   - The instantaneous spot rate (short rate) approximately.

3. **(Bonds and Arbitrage)**
Suppose the bond market is deterministic and we have
$$P(t,S) < P(t,T)P(T,S) \qquad \text{for some } t \le T \le S.$$
Show that there is an arbitrage opportunity.

4. **(Caps, Floors and Bond-Options)**
Let $0 < T < S$ be two fixed time instants and $\tau(T,S)$ the time between T and S. $K > 0$ denotes the strike price and $N > 0$ the nominal value. Show that a caplet with time S-payoff
$$N\tau(T,S)\,(L(T,S)-K)^+$$
can be interpreted as a put on a zero-coupon bond. Thus, a cap is a portfolio of put options on bonds, whereas a floor is a portfolio of call options on bonds.
Hint: Consider the present value of the caplet payoff at time S.

5. **(Put-Call Parity for Caps and Floors)**
Show that put-call parity for caps and floors holds, i.e., show that
$$\text{Cap}(t,\mathcal{T},N,K) - \text{Flr}(t,\mathcal{T},N,K) = \text{PFS}(t,\mathcal{T},N,K),$$
$$\text{PS}(t,\mathcal{T},N,K) - \text{RS}(t,\mathcal{T},N,K) = \text{PFS}(t,\mathcal{T},N,K),$$
where $\text{Cap}(t,\mathcal{T},N,K)$ denotes the price of the cap at time t, $\text{Flr}(t,\mathcal{T},N,K)$ the price of the floor at time t, $\text{PS}(t,\mathcal{T},N,K)$ and $\text{RS}(t,\mathcal{T},N,K)$ the prices of a payer swaption and a receiver swaption at time t and $\text{PFS}(t,\mathcal{T},N,K)$ denotes the price of a payer IRS at time t.