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Winter Term 10/11

Institute of Mathematical Finance

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Fixed Income Models

Exercise Sheet 6

(Due: Tuesday 12/07/2010)

33. **(Trinomial Tree)**
Read http://www.rotman.utoronto.ca/∼amackay/fin/USETREE1a.pdf and use a trinomial tree to compute the price of 5%-coupon bond with a maturity of 4 years in the

    (a) Black–Karasinski model with $k = 0.2$, $\sigma = 0.22$ and $\theta = -3$,

    (b) Hull–White model with parameters $k = 0.1$, $\theta = 0.05$, $\sigma = 0.15$,

    (c) CIR++ model with parameters $k = 0.1$, $\theta = 0$, $\sigma = 0.15$ and $\varphi(t) = 0.05 + 0.0001t$.

34. **(Simulation in the CIR Framework)**
Recall the CIR model with risk-neutral dynamics

$$\mathrm{d}r(t) = k\left(\theta - r(t)\right)\mathrm{d}t + \sigma\sqrt{r(t)}\mathrm{d}W(t), \quad r(0) = r_0.$$

Suppose the parameters of the model are $r_0 = 0.02$, $k = 1$, $\theta = 0.04$, $\sigma = 0.15$. Use Euler's scheme to complete the following exercises:

    (a) Simulate a path of $r(t)$, $t \in [0, T]$, $T = 5$. Let us interpret t as time measured in years. Choose a time step of one month. Plot your results.

    (b) Analyse, whether Euler's scheme is a good way to simulate a CIR process (does the simulated process possess all the properties of the original process?).

    (c) Consider a zero-coupon bond expiring in $T = 5$. Find the price of this bond for each time t_i for which you simulated $r(t)$, i.e., find $P(t_i, 5)$ for $i = 0, \ldots, n$. Find the corresponding yield $R(t_i, 5)$. Plot price and yield. Compare $R(t, 5)$ and $r(t)$. What do you see?

    (d) Find the term structure for each month, i.e., find for each $r(t_i)$ the values $R(t_i, t_{i+s})$ for $s = 1, 2, \ldots, 10$ years. Plot your results in a 3D Plot, i.e., plot the function $(t, s) \mapsto R(t, t + s)$.

35. **(Moraleda and Vorst Model)**
The Moraleda and Vorst model is given by the following dynamics:

$$\mathrm{d}r(t) = \left(\theta(t) - \beta(t)r(t)\right)\mathrm{d}t + \sigma\mathrm{d}W(t), \text{ where } \beta(t) = \lambda - \frac{\gamma}{1+\gamma t},$$

$\sigma > 0$, θ deterministic, and $\lambda, \gamma \in \mathbb{R}$.

    (a) Give $r(t)$ explicitely by solving the SDE.

    (b) Find $\mathbb{E}\left(r(t)|\mathcal{F}(s)\right)$ and $\mathbb{V}\left(r(t)|\mathcal{F}(s)\right)$.

    (c) Find $P(t, T)$.

    (d) Find $\mathrm{d}P(t, T)$ and $\mathrm{d}\frac{1}{P(t,T)}$.

    (e) Find $f(t, T)$ and $\mathrm{d}f(t, T)$.

    (f) Show that

$$\sqrt{\frac{\mathbb{V}\mathrm{d}f(t,T)}{\mathrm{d}t}} = \sigma\frac{1+\gamma T}{1+\gamma t}e^{-\lambda(T-t)}.$$

    (g) Find $\mathrm{d}W^T(t)$ and the dynamics of r under $\mathbb{Q}^T$.

    (h) Find $\mathrm{ZBC}(t, T, S, K)$.