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Winter Term 10/11

Institute of Mathematical Finance

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Fixed Income Models

Exercise Sheet 7

(Due: Tuesday 01/11/2011)

36. **(Two-Factor G2++ Model, Independent Brownian Motions)**
Consider the G2++ model as introduced in the lecture. Let $\tilde{W}_1$ and $\tilde{W}_2$ be two independent Brownian motions. Define $\mathrm{d}W_1 := \mathrm{d}\tilde{W}_1$ and $\mathrm{d}W_2$ as a linear combination of $\mathrm{d}\tilde{W}_1$ and $\mathrm{d}\tilde{W}_2$ such that $\mathrm{d}W_1\mathrm{d}W_2 = \rho\mathrm{d}t$. Compute the short rate in this setting.

37. **(Correlation of Short and Long Rate)**
Compute the correlation of the short and long rate (compare Remark 6.1) in

    (a) the G2++ model,

    (b) the Hull–White two-factor model,

    (c) the Longstaff–Schwartz model,

    (d) the CIR2++ model.

38. **(HJM)**
Consider the trivial HJM model where the forward rates $f(t,T)$ are deterministic. Show that all forward measures collapse into the risk-neutral measure, and show that forward prices equal futures prices, where the forward price is defined as

$$\mathrm{For}(t,T) = \frac{f(t,T)}{B(t,T)}.$$

39. **(Parallel Shift of Forward Curve, Arbitrage)**
Consider first the one-period model for the forward curve

$$f(0,t) = 0.04, \qquad t \geq 0,$$

$$f(\omega,1,t) = \begin{cases} 0.06, & t \geq 1, \quad \omega = \omega_1, \\ 0.02, & t \geq 1, \quad \omega = \omega_2, \end{cases}$$

where $\omega = \{\omega_1, \omega_2\}$ and $\mathbb{P}(\omega_i) > 0$, $i = 1, 2$.

    (a) Show that the matrix

$$\begin{pmatrix} P(0,1) & P(0,2) & P(0,3) \\ P(\omega_1,1,1) & P(\omega_1,1,2) & P(\omega_1,1,3) \\ P(\omega_2,1,1) & P(\omega_2,1,2) & P(\omega_2,1,3) \end{pmatrix}$$

    is invertible.

    (b) Use (a) to find an arbitrage strategy with value process $V(0) = 0$ and $V(\omega_i, 1) = 1$ for both ω_i.

    (c) For some deterministic initial curve $f(0,T) = h(T)$ and some Itô process $\mathrm{d}Z(t) = b(t)\mathrm{d}t + \rho(t)\mathrm{d}W(t)$, define the parallel shift of the forward curve by

$$f(t,T) = h(T-t) + Z(t).$$

    Show that the HJM drift restriction implies $b(t) \equiv b$, $\rho^2(t) \equiv a$, and

$$h(x) = -\frac{a}{2}x^2 + bx + c$$

    for some constants $a \geq 0$ and $b, c \in \mathbb{R}$.

40. **(HJM, Hull–White Model)**
Prove the statements given in Remark 7.24.

41. **(HJM, Ho–Le Model)**
Prove the statements given in Remark 7.27.

42. **(Mercurio–Moraleda Model, ZBC Option)**
Use Theorem 7.29 to prove Theorem 7.32.

43. **(Mercurio–Moraleda Model, Futures Contract)**
Use Theorem 7.30 to prove Theorem 7.33.

Merry Christmas and a happy new year!