## MISSOURI

## Math 1215, Exam 3

## Preparation Package

Exam 3 will be on Thursday, April 21, 5 - 5:50 pm.
Please check your room assignments below: Labs 301, 305, 306, and 310 (Wang) - BCH 120 Labs 302, 303, 307, and 308 (Kovach) - H-SS G5 Labs 304, 309, 311, and 312 (Yuan) - McNutt 204

For DSS students, be sure to be in contact with the Testing Center. Exams will be taken at the same day and time (plus some possible extra time) as the regularly scheduled exam.

If you are sick or in quarantine, e-mail me no later than 4:30 pm on the day before the exam to arrange for alternate testing accommodation at the same day and time as the regularly scheduled exam.

This preparation package contains, besides the information on this page, a list of things that you should know, and a practice exam that features the exact instructions and the same formula sheet as the one on the real exam as well as ten practice problems that are similar to the ones on the real exam. Please work through them. The review on Wednesday will consist of an asynchronous zoom posting of me working out these practice problems.


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## Math 1215, Exam 3

## You should be able to do all of the following

1. Be able to do everything that was covered in Exam 1 and Exam 2.
2. Know about sequences, series, and partial sums (Sections $10.1-3$ ).
3. Find the sum of a convergent geometric series (Section 10.3).
4. Use the Divergence Test (DT) to determine divergence of a series (Section 10.4).
5. Use the Integral Test (IT) to determine convergence or divergence of a series (Section 10.4).
6. Use the Comparison Test (CT) and the Limit Comparison Test (LCT) to determine convergence or divergence of a series (Section 10.5).
7. Use the Alternating Series Test (AST) to determine convergence of a series. Be able to determine whether a series with alternating terms is convergent, absolutely convergent, or conditionally convergent (Section 10.6).
8. Use the Ratio Test (RaT) and the Root Test (RoT) to determine absolute convergence or divergence of a series (Section 10.7).
9. Know how to find the $n$th Taylor polynomial of a given function around a given center (Section 11.1).
10. Find the radius of convergence and the interval of convergence of a power series (Section 11.2).

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## Math 1215, Practice Exam 3

## Please print your name in this box

## Instructions

1. Be sure to clearly print your name in the space provided at the top of each page.
2. No calculators, books, or other materials are permitted.
3. This exam has 6 sheets of paper (front and back). Do not remove the staple! There are 100 points. Each of the ten problems is 10 points. Once this exam starts, you have 50 minutes. This means you have about 5 minutes for each of the 10 problems.
4. You must write darkly and legibly - this exam will be scanned for electronic grading.
5. Work all problems. Show all work. Full credit will be given only if work is shown which fully justifies your answer.
6. There will be sufficient space under each problem in which to show your work. No extra paper is allowed.
7. Place each final answer in the provided box. All final answers must be simplified!
8. Turn off your cell phone if you have one with you.

Do not turn this page until told to do so.


## Potentially useful Facts.

| Geometric <br> Series | $\sum_{n=0}^{\infty} r^{n}$ | convergent to $\frac{1}{1-r}$ if $-1<r<1$ otherwise divergent |
| :---: | :---: | :---: |
| Harmonic Series | $\sum_{n=1}^{\infty} \frac{1}{n}$ | divergent |
| $p$-Series | $\sum_{n=1}^{\infty} \frac{1}{n^{p}}$ | convergent if $p>1$ otherwise divergent |
| Telescoping Series | $\sum_{n=1}^{\infty}\left(c_{n}-c_{n+1}\right)$ | convergent if $\lim _{n \rightarrow \infty} c_{n}$ exists otherwise divergent |
| Divergence Test | $\sum_{n=1}^{\infty} a_{n}$ | divergent if $\lim _{n \rightarrow \infty} a_{n} \neq 0$ |
| Integral Test | $\begin{gathered} \sum_{n=1}^{\infty} f(n) \\ f \text { continuous, } \\ \text { positive, } \\ \text { decreasing } \\ \hline \end{gathered}$ | convergent if $\int_{1}^{\infty} f(x) \mathrm{d} x$ is convergent divergent if $\int_{1}^{\infty} f(x) \mathrm{d} x$ is divergent |
| Comparison Test | $\sum_{\substack{n=1 \\ a_{n}, b_{n}>0}}^{\infty} a_{n}$ | convergent if $a_{n} \leq b_{n}$ and $\sum_{n=1}^{\infty} b_{n}$ converges divergent if $b_{n} \leq a_{n}$ and $\sum_{n=1}^{\infty} b_{n}$ diverges |
| $\begin{aligned} & \text { Limit } \\ & \text { Comparison } \\ & \text { Test } \end{aligned}$ | $\sum_{\substack{n=1 \\ a_{n}, b_{n}>0}}^{\infty} a_{n}$ | convergent if $\lim _{n \rightarrow \infty} \frac{a_{n}}{b_{n}}<\infty$ and $\sum_{n=1}^{\infty} b_{n}$ converges divergent if $\lim _{n \rightarrow \infty} \frac{a_{n}}{b_{n}}>0$ and $\sum_{n=1}^{\infty} b_{n}$ diverges |
| Alternating Series Test | $\sum_{n=1}^{\infty}(-1)^{n-1} b_{n}$ | convergent if $b_{n}>0$ are nonincreasing and converge to zero |
| Ratio Test | $\sum_{n=1}^{\infty} a_{n}$ | absolutely convergent if $\lim _{n \rightarrow \infty}\left\|\frac{a_{n+1}}{a_{n}}\right\|<1$ divergent if $\lim _{n \rightarrow \infty}\left\|\frac{a_{n+1}}{a_{n}}\right\|>1$ |
| Root Test | $\sum_{n=1}^{\infty} a_{n}$ | absolutely convergent if $\lim _{n \rightarrow \infty} \sqrt[n]{\left\|a_{n}\right\|}<1$ divergent if $\lim _{n \rightarrow \infty} \sqrt[n]{\left\|a_{n}\right\|}>1$ |
| Power Series | $\sum_{n=0}^{\infty} c_{n}(x-a)^{n}$ | radius of convergence can be $R=0, R=\infty$, or $0<R<\infty$ |
| Taylor <br> Polynomial | $\sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^{k}$ | matches $f$ at all $f^{(k)}(a), 0 \leq k \leq n$ |

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Practice Problem Number I.
Let $a_{0}=a_{1}=1$ and define $a_{n+2}=2 a_{n}+a_{n+1}$ recursively for $n \geq 0$. Find $a_{5}$.


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## Practice Problem Number 2.

Let $a_{n}=\frac{60(-1)^{n-1}}{n}$ and $S_{n}=\sum_{k=1}^{n} a_{k}$ for $n \geq 1$.
Find $S_{5}$.


Please print your name in this box

Practice Problem Number 3. Find the infinite sum of the shaded areas.


## Missouri Please print your name in this box

## Practice Problem Number 4.

## Consider the options

Now put in the first box below the letter that describes the series

$$
\sum_{n=1}^{\infty} \frac{1}{n^{\pi}}
$$

put in the middle box below the letter that describes the series

$$
\sum_{n=1}^{\infty} \frac{(-1)^{n} \pi^{n}}{e^{n}}
$$

and put in the last box below the letter that describes the series

$$
\sum_{n=1}^{\infty} \frac{(-1)^{n}}{\sqrt[\pi]{n}}
$$

## Answer:

(one character per box)

## MISSOURI \cline { 2 - 3 } name in this box

Practice Problem Number 5.
Determine whether $\sum_{n=1}^{\infty} \frac{2 n^{2}-1}{3 n^{2}+4}$ is convergent or divergent.

## Practice Problem Number 6.

Use the integral test to determine whether $\sum_{n=2}^{\infty} \frac{n}{n^{2}-1}$ is convergent or divergent.


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Practice Problem Number 7.
Determine whether the series $\sum_{n=2}^{\infty} \frac{(-1)^{n-1} n}{n^{2}-1}$ is convergent or divergent.

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Practice Problem Number 8.
Use any test other than the integral test to determine whether the series $\sum_{n=2}^{\infty} \frac{n}{n^{2}-1}$ is convergent or divergent.

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## Practice Problem Number 9.

Let $f(x)=x e^{x}$. Find $p_{3}(0.1)$, where $p_{3}$ is the Taylor polynomial of order 3 of $f$ centered at 0 .


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Practice Problem Number 10.
Find the radius of convergence of $\sum_{n=1}^{\infty}\left(\frac{n^{3}+1}{4 n^{3}+1}\right)^{n} x^{n}$.

