MISSOURI S&T

Math 1215, Final Exam Preparation Package

The Final Exam will be on Thursday, May 12, 7:30 – 9:30 am.

Please check your room assignments below:
Labs 301, 305, 306, and 310 (Wang) - BCH 120
Labs 302, 303, 307, and 308 (Kovach) - CS 120
Labs 304, 309, 311, and 312 (Yuan) - CS 120

For DSS students, be sure to be in contact with the Testing Center. Exams will be taken at the same day and time (plus some possible extra time) as the regularly scheduled exam.

If you are sick or in quarantine, e-mail me no later than 4:30 pm on the day before the exam to arrange for alternate testing accommodation at the same day and time as the regularly scheduled exam.

This preparation package contains, besides the information on this page, a list of things that you should know, and a practice exam that features the exact instructions and the same formula sheet as the one on the real exam as well as 63 practice that include the problems on the real exam. Please work through them. The review on Friday will consist of an asynchronous zoom posting of me working out these practice problems.

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Math 1215, Final Exam.

You should be able to do all of the following

1. Be able to do all ten problems from Practice Exam 1
   (Practice Problems 1-10)
2. Be able to do all ten problems from Exam 1.
   (Practice Problem 11-20)
3. Be able to do all ten problems from Practice Exam 2.
   (Practice Problems 21-30)
4. Be able to do all ten problems from Exam 2.
   (Practice Problems 31-40)
5. Be able to do all ten problems from Practice Exam 3.
   (Practice Problems 41-50)
6. Be able to do all ten problems from Exam 3.
   (Practice Problems 51-60)
7. Be able to do the three practice problems for Chapter 12.
   (Practice Problems 61-63)

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Math 1215, Final Exam

Please print your name in this box	

Instructions

1. Be sure to clearly print your name in the space provided at the top of each page.
2. No calculators, books, or other materials are permitted.
3. This exam has 11 sheets of paper (front and back). *Do not remove the staple!* There are 100 points. Each of the 20 problems is 5 points. Once this exam starts, you have 120 minutes. This means you have about 6 minutes for each of the 20 problems.
4. You must write darkly and legibly – this exam will be scanned for electronic grading.
5. Work all problems. Show all work. Full credit will be given only if work is shown which fully justifies your answer.
6. There will be sufficient space under each problem in which to show your work. No extra paper is allowed.
7. Place each final answer in the provided box. *All final answers must be simplified!*
8. **Turn off your cell phone if you have one with you.**

Do not turn this page until told to do so.

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\,\mathrm{d}x$$

$$S = \int_a^b 2\pi f(x)\sqrt{1 + (f'(x))^2}\,\mathrm{d}x$$

$$\left(f^{-1}\right)'(x) = \frac{1}{f'\left(f^{-1}(x)\right)}$$

$$\int \frac{\mathrm{d}x}{\sqrt{1 - x^2}} = \sin^{-1}(x) + c$$

$$\int \frac{\mathrm{d}x}{1 + x^2} = \tan^{-1}(x) + c$$

$$\int \frac{\mathrm{d}x}{x\sqrt{x^2 - 1}} = \sec^{-1}|x| + c$$

$$\cos\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$

$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

Geometric Series	$\sum_{n=0}^{\infty} r^n$	convergent to $\frac{1}{1-r}$ if $-1 < r < 1$ otherwise divergent
Harmonic Series	$\sum_{n=1}^{\infty} \frac{1}{n}$	divergent
p-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	convergent if $p > 1$ otherwise divergent
Telescoping Series	$\sum_{n=1}^{\infty} \left(c_n - c_{n+1}\right)$	convergent if $\lim_{n\to\infty} c_n$ exists otherwise divergent
Divergence Test	$\sum_{n=1}^{\infty} a_n$	divergent if $\lim_{n\to\infty} a_n \neq 0$
Integral Test	$\sum_{n=1}^{\infty} f(n)$ f continuous, positive, decreasing	convergent if $\int_1^{\infty} f(x)\mathrm{d}x$ is convergent divergent if $\int_1^{\infty} f(x)\mathrm{d}x$ is divergent
Comparison Test	$\sum_{n=1}^{\infty} a_n$ $a_n, b_n > 0$	convergent if $a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges divergent if $b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges
Limit Comparison Test	$\sum_{n=1}^{\infty} a_n$ $a_n, b_n > 0$	convergent if $\lim_{n\to\infty} \frac{a_n}{b_n} < \infty$ and $\sum_{n=1}^{\infty} b_n$ converges divergent if $\lim_{n\to\infty} \frac{a_n}{b_n} > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges
Alternating Series Test	$\sum_{n=1}^{\infty} (-1)^{n-1} b_n$	convergent if $b_n > 0$ are nonincreasing and converge to zero
Ratio Test	$\sum_{n=1}^{\infty} a_n$	absolutely convergent if $\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert < 1$ divergent if $\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert > 1$
Root Test	$\sum_{n=1}^{\infty} a_n$	absolutely convergent if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} < 1$ divergent if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} > 1$
Power Series	$\sum_{n=0}^{\infty} c_n \left(x - a\right)^n$	radius of convergence can be $R = 0$, $R = \infty$, or $0 < R < \infty$
Taylor Polynomial	$\sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!} \left(x - a\right)^k$	matches f at all $f^{(k)}(a)$, $0 \leq k \leq n$

$$M(n) = [f(m_1) + f(m_2) + \ldots + f(m_n)]\Delta x$$

$$T(n) = \left[\frac{f(x_0)}{2} + f(x_1) + f(x_2) + \ldots + f(x_{n-1}) + \frac{f(x_n)}{2}\right]\Delta x$$

$$S(n) = [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 4f(x_{n-1}) + f(x_n)]\frac{\Delta x}{3}$$

$$A = \frac{1}{2}\int_\alpha^\beta [f(\theta)]^2\,\mathrm{d}\theta, \quad L = \int_\alpha^\beta \sqrt{[f(\theta)]^2 + [f'(\theta)]^2}\,\mathrm{d}\theta$$

Please print your name in this box	

Practice Problem Number 1.

Find the volume of the solid formed by revolving the region bounded by $f(x) = \sqrt{x}$ and $g(x) = x$ about the x-axis.

Answer:
(one character per box)

$$\frac{\square}{\square}$$

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Practice Problem Number 2.

Find the volume of the solid formed by revolving the region bounded by $y = 1 - x$ and the x-axis in the first quadrant about the y-axis.

Answer:
(one character per box)

$$\frac{\square}{\square}$$

Please print your name in this box

Practice Problem Number 3.

Find the arc length of the curve $x = \frac{2}{3}(y-1)^{3/2}$ for $1 \leq y \leq 4$.

Answer:
(one character per box)

$$\frac{\square\,\square}{\square}$$

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Practice Problem Number 4.

Find the area of the surface generated when the graph of $y = \sqrt{9 - x^2}$ for $-2 \le x \le 2$ is rotated about the x-axis.

Answer:
(one character per box)

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Practice Problem Number 5.

If $f(x) = x^2 + e^x$, find $(f^{-1})'(1)$.

Answer:
(one character per box)

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Practice Problem Number 6.

Find the integral $\displaystyle\int_e^{e^2} \frac{dx}{x \ln^2(x)}$.

Answer:
(one character per box)

$$\frac{\square}{\square}$$

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Practice Problem Number 7.

Find the integral $\displaystyle\int_3^4 4^x \mathrm{d}x.$

Answer:
(one character per box)

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Practice Problem Number 8.

Consider the intervals

(A) $$(-\infty, \infty)$$

(B) $$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

(C) $$[-1, 1]$$

(D) $$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

(E) $$[0, \pi].$$

Now put in the first box below the letter that corresponds to the domain of the inverse sine, put in the middle box below the letter that corresponds to the domain of the inverse tangent, and put in the last box below the letter that corresponds to the domain of the inverse cosine.

Answer:
(one character per box)

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Practice Problem Number 9.

Find the integral $$\int_0^{\frac{1}{4}} \frac{dx}{\sqrt{\frac{1}{16} - x^2}}.$$

Answer:
(one character per box)

$$\frac{\square}{\square}$$

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Practice Problem Number 10.

Evaluate $\lim_{x \to 0^+} \sqrt{x} \ln(x)$.

Answer: ☐

(one character per box)

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Practice Problem Number 11.

Find the volume of the solid formed by revolving the region in the first quadrant bounded by $y = \sqrt{2x}$ and $x = 1$ about the x-axis.

Answer:
(one character per box)

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Practice Problem Number 12.

Find the volume of the solid formed by revolving the region bounded by $y = 1/\pi$, $y = 0$, $x = 0$, and $x = 1$ about the y-axis.

Answer:

(one character per box)

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Practice Problem Number 13.

Find the arc length of the curve $y = \ln(\sec x)$ for $0 \le x \le \pi/4$.

Answer: $\square\,\square\left(\sqrt{\square}\,\square\,\square\right)$

(one character per box)

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Please print your name in this box	

Practice Problem Number 14.

Find the area of the surface generated when the graph of $y = \sqrt{1 - x^2}$ for $0 \leq x \leq 1/2$ is rotated about the x-axis.

Answer:
(one character per box)

$\square$

Please print your name in this box

Practice Problem Number 15.

If $f(x) = 3 + x + e^x$, find $(f^{-1})'(4)$.

Answer:
(one character per box)

$$\frac{\square}{\square}$$

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Practice Problem Number 16.

Find the integral $\displaystyle\int_{e}^{e^2} \frac{\mathrm{d}x}{x\ln(x)}$.

Answer:
(one character per box)

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Please print your name in this box	

Practice Problem Number 17.

Find the integral $\int_1^2 5^x \, \mathrm{d}x$.

Answer:
(one character per box)

Please print your name in this box	

Practice Problem Number 18.

Consider the intervals

(A) $$[\mathbf{0}, \pi]$$

(B) $$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

(C) $$[-\mathbf{1}, \mathbf{1}]$$

(D) $$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

(E) $$(-\infty, \infty).$$

Now put in the first box below the letter that corresponds to the range of the inverse sine, put in the middle box below the letter that corresponds to the range of the inverse cosine, and put in the last box below the letter that corresponds to the range of the inverse tangent.

Answer:
(one character per box)

Please print your name in this box	

Practice Problem Number 19.

Find the integral $\displaystyle\int_0^{\frac{1}{4}} \frac{\mathrm{d}x}{\frac{1}{16}+x^2}$.

Answer:
(one character per box)

Please print your name in this box

Practice Problem Number 20.

Evaluate $\lim_{x \to 0} \dfrac{e^{4x} - 1 - 4x}{x^2}$.

Answer:
(one character per box)

$\square$

Please print your name in this box	

Practice Problem Number 21.

Simplify $\ln\left(\cosh(2) + \sinh(2)\right)$.

Answer:
(one character per box)

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Practice Problem Number 22.

Find the integral $\displaystyle\int_{-3}^{-2} \frac{dx}{x^2+6x+10}$.

Answer:
(one character per box)

$$\frac{\square}{\square}$$

Please print your name in this box

Practice Problem Number 23.

Find the integral $\int_0^{\ln(2)} x \sinh(x) \mathrm{d}x$.

Answer:
(one character per box)

$$\frac{\square\,\square\,\square\,\square - \square}{\square}$$

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Please print your name in this box	

Practice Problem Number 24.

Consider the u-substitutions

(A) $$u = \cos(x)$$

(B) $$u = \sec(x)$$

(C) $$u = \sin(x)$$

(D) $$u = \tan(x).$$

Now put in the first box below the letter that corresponds to the u-substitution that should be done to evaluate the integral

$$\int \sin^6(x)\cos^9(x)\mathrm{d}x,$$

put in the middle box below the letter that corresponds to the u-substitution that should be done to evaluate the integral

$$\int \sin^7(x)\cos^8(x)\mathrm{d}x,$$

and put in the last box below the letter that corresponds to the u-substitution that should be done to evaluate the integral

$$\int \sec^8(x)\tan^4(x)\mathrm{d}x.$$

Answer:
(one character per box)

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Practice Problem Number 25.

Find the integral $\displaystyle\int_1^2 \sqrt{2x - x^2}\,\mathrm{d}x.$

Answer:
(one character per box)

Please print your name in this box

Practice Problem Number 26.

Find the integral $\displaystyle\int_2^3 \frac{2}{x^2-1}\,dx.$

Answer:
(one character per box)

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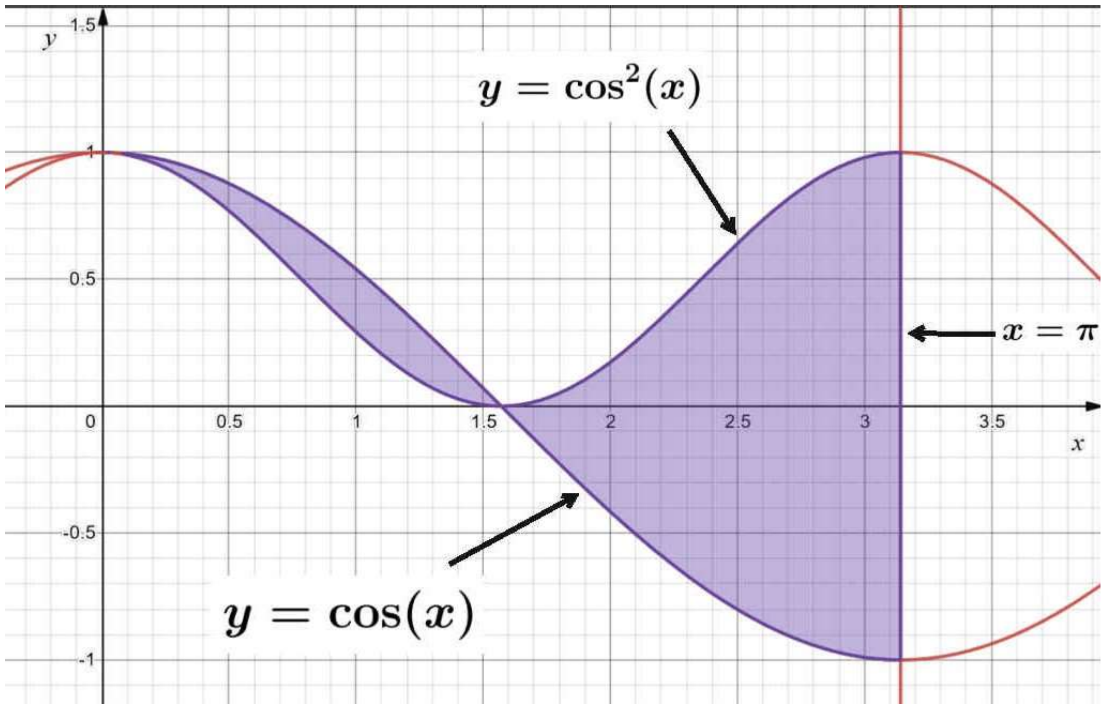
Practice Problem Number 27.

Calculate the shaded area.



Answer:
(one character per box)

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Practice Problem Number 28.

Find the integral $\displaystyle\int_1^2 \frac{\mathrm{d}x}{1+\sqrt{x-1}}$.

Answer:
(one character per box)

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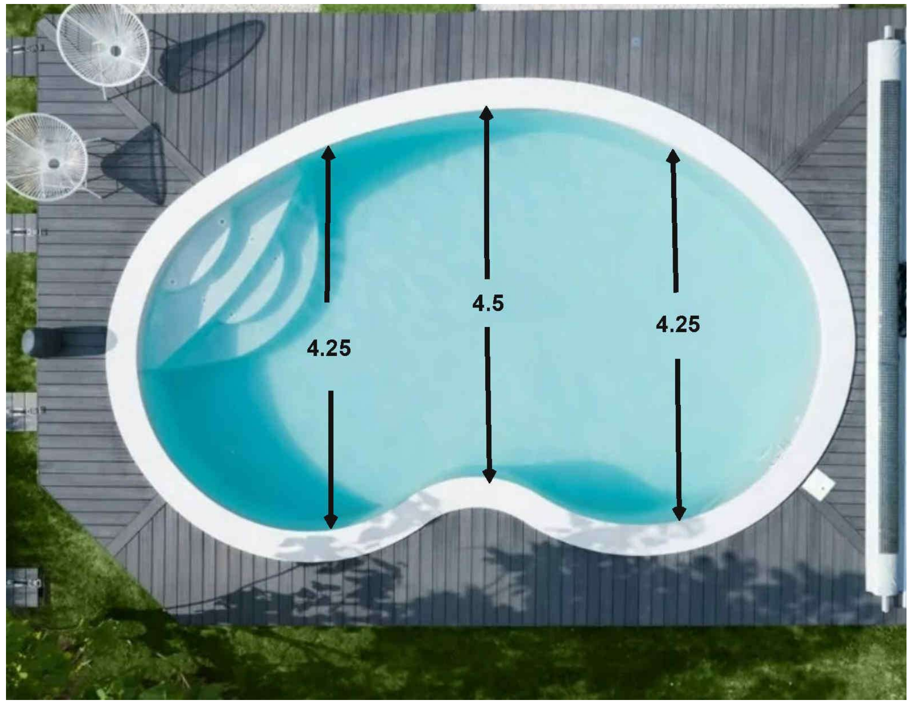
Practice Problem Number 29.



The widths (in meters) of a kidney-shaped swimming pool were measured at 2-meter intervals as indicated in the picture. Use the trapezoid rule to estimate the area of the swimming pool (in square meters).

Answer:
(one character per box)

Please print your name in this box	

Practice Problem Number 30.

Evaluate the improper integral $\displaystyle\int_0^1 \frac{\mathrm{d}x}{\sqrt{x}}$.

Answer:
(one character per box)

Please print your name in this box

Practice Problem Number 31.

Find the arc length of the curve $y = \cosh(x)$ for $0 \leq x \leq \ln(2)$.

Answer:
(one character per box)

$$\frac{\square}{\square}$$

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Practice Problem Number 32.

Find the integral $\displaystyle\int_0^1 \frac{4\,\mathrm{d}x}{x^2 - 2x + 2}$.

Answer:
(one character per box)

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Practice Problem Number 33.

Find the integral $\displaystyle\int_{\pi}^{2\pi} \frac{x\cos(x)\mathrm{d}x}{2}$.

Answer:
(one character per box)

$\square$

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Practice Problem Number 34.

Consider the u-substitutions

(A) $$u = \sin(x)$$

(B) $$u = \cos(x)$$

(C) $$u = \sec(x)$$

(D) $$u = \tan(x).$$

Now put in the first box below the letter that corresponds to the u-substitution that should be done to evaluate the integral

$$\int \sin^3(x)\cos^2(x)\mathrm{d}x,$$

put in the middle box below the letter that corresponds to the u-substitution that should be done to evaluate the integral

$$\int \sin^2(x)\cos^3(x)\mathrm{d}x,$$

and put in the last box below the letter that corresponds to the u-substitution that should be done to evaluate the integral

$$\int \sec^2(x)\tan^4(x)\mathrm{d}x.$$

Answer:
(one character per box)

Please print your name in this box	

Practice Problem Number 35.

Find the integral $\int_0^1 4\sqrt{1-x^2}\,dx$**.**

Answer:
(one character per box)

Please print your name in this box	

Practice Problem Number 36.

Find the integral $$\int_0^1 \frac{dx}{x^2+3x+2}.$$

Answer:
(one character per box)

Please print your name in this box

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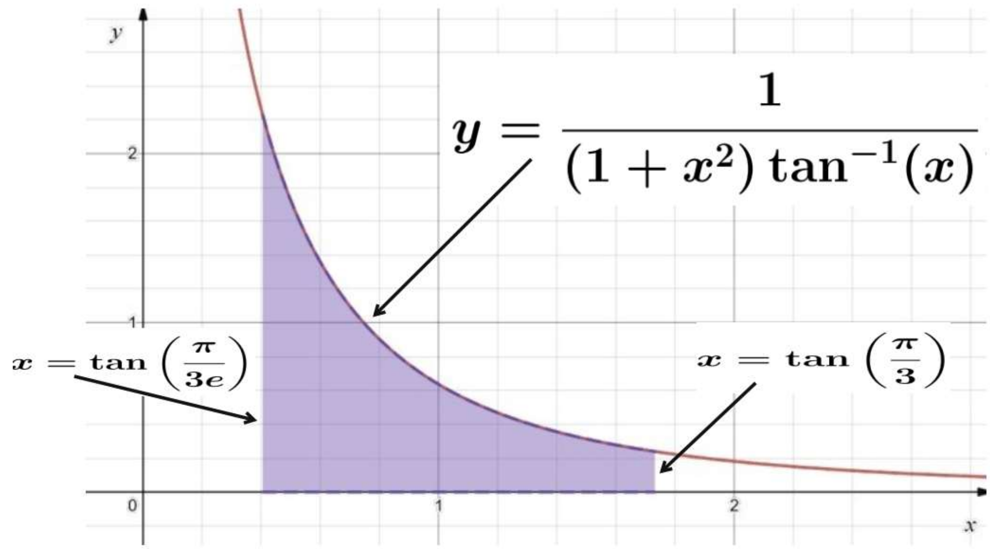
Practice Problem Number 37.

Calculate the shaded area.



Answer:
(one character per box)

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Practice Problem Number 38.

Find the integral $$\int_{1/e}^{1} \frac{4\,\mathrm{d}x}{x(\ln^2(x) + 2\ln(x) + 2)}.$$

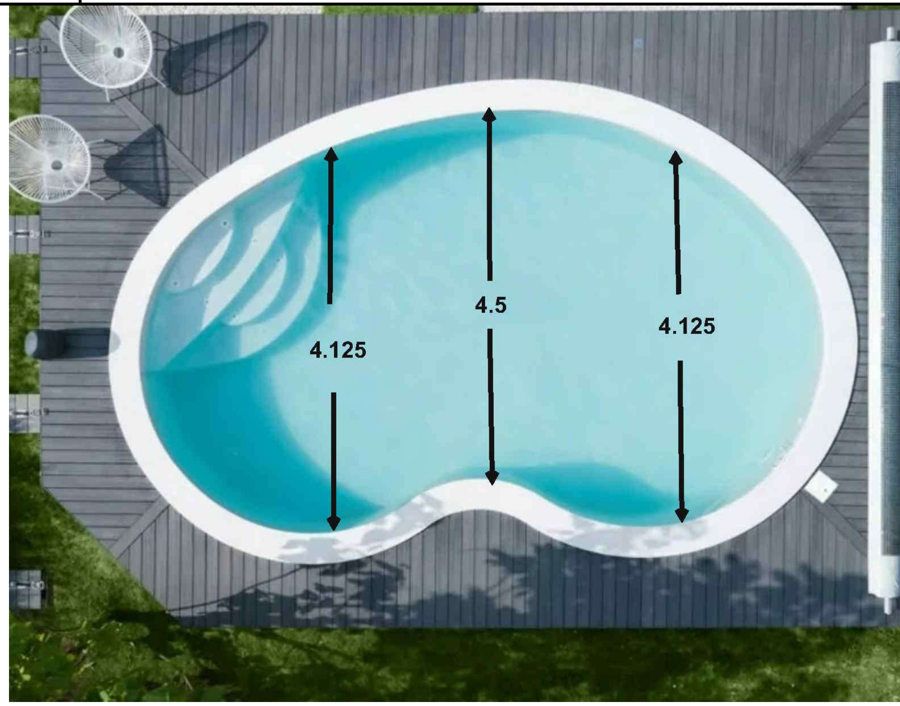
Answer:
(one character per box)

$\square$

Please print your name in this box	

Practice Problem Number 39.

The widths (in meters) of a kidney-shaped swimming pool were measured at 2-meter intervals as indicated in the picture. Use Simpson's rule to estimate the area of the swimming pool (in square meters).



Answer:
(one character per box)

Please print your name in this box

Practice Problem Number 40.

Evaluate the improper integral $\displaystyle\int_1^\infty \frac{dx}{x^2}$.

Answer:
(one character per box)

Please print your name in this box

Practice Problem Number 41.

Let $a_0 = a_1 = 1$ and define $a_{n+2} = 2a_n + a_{n+1}$ recursively for $n \geq 0$. Find a_5.

Answer:
(one character per box)

Please print your name in this box	

Practice Problem Number 42.

Let $a_n = \dfrac{60(-1)^{n-1}}{n}$ **and** $S_n = \displaystyle\sum_{k=1}^{n} a_k$ **for** $n \geq 1$.

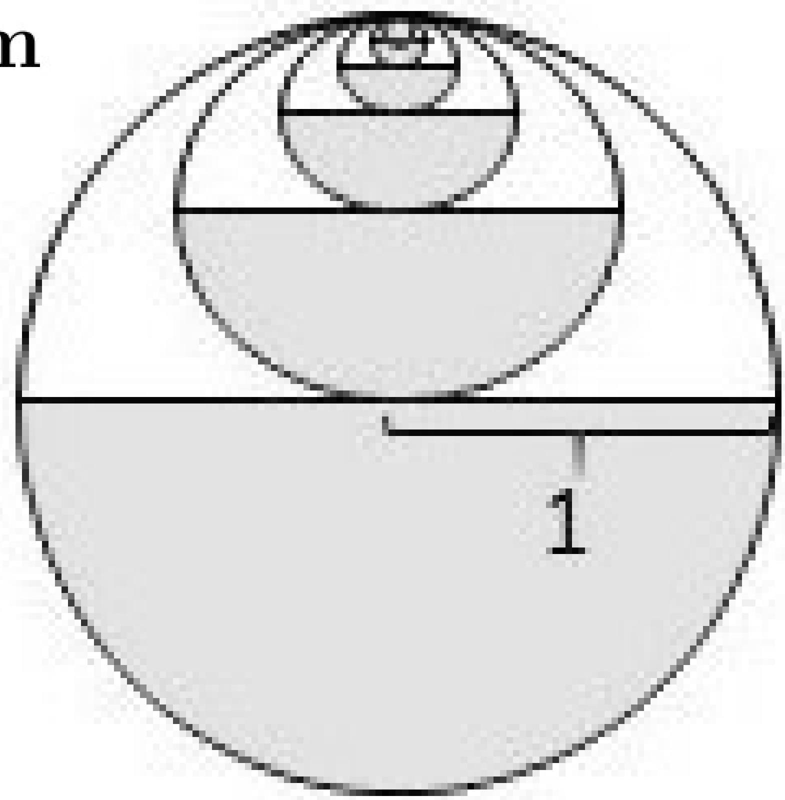
Find S_5.

Answer:
(one character per box)

Please print your name in this box	

Practice Problem Number 43.

Find the infinite sum of the shaded areas.



Answer:
(one character per box)

Please print your name in this box	

Practice Problem Number 44.

Consider the options

(A) **series is absolutely convergent**

(C) **series is conditionally convergent**

(D) **series is divergent.**

Now put in the first box below the letter that describes the series

$$\sum_{n=1}^{\infty} \frac{1}{n^{\pi}},$$

put in the middle box below the letter that describes the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n \pi^n}{e^n},$$

and put in the last box below the letter that describes the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[\pi]{n}}.$$

Answer:
(one character per box)

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Practice Problem Number 45.

Determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2-1}{3n^2+4}$ **is convergent or divergent.**

Answer: $\square$

(put C for convergent or D for divergent)

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Practice Problem Number 46.

Use the integral test to determine whether $\displaystyle\sum_{n=2}^{\infty} \frac{n}{n^2-1}$ **is convergent or divergent.**

Answer:
(put C for convergent or D for divergent)

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Please print your name in this box	

Practice Problem Number 47.

Determine whether the series $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n-1} n}{n^2 - 1}$ is convergent or divergent.

Answer:

(put C for convergent or D for divergent)

$\square$

Please print your name in this box	

Practice Problem Number 48.

Use any test other than the integral test to determine whether the series $\sum_{n=2}^{\infty} \frac{n}{n^2-1}$ is convergent or divergent.

Answer:
(put C for convergent or D for divergent)

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Practice Problem Number 49.

Let $f(x) = xe^x$. Find $p_3(0.1)$, where p_3 is the Taylor polynomial of order 3 of f centered at 0.

Answer:
(one character per box)

Please print your name in this box

Practice Problem Number 50.

Find the radius of convergence of $\sum_{n=1}^{\infty} \left(\frac{n^3+1}{4n^3+1} \right)^n x^n.$

Answer:

(one character per box)

Please print your name in this box

Practice Problem Number 51.

Let $a_0 = a_1 = 2$ and define $a_{n+2} = a_n + a_{n+1}$ recursively for $n \geq 0$. Find a_4.

Answer:
(one character per box)

Please print your name in this box

Practice Problem Number 52.

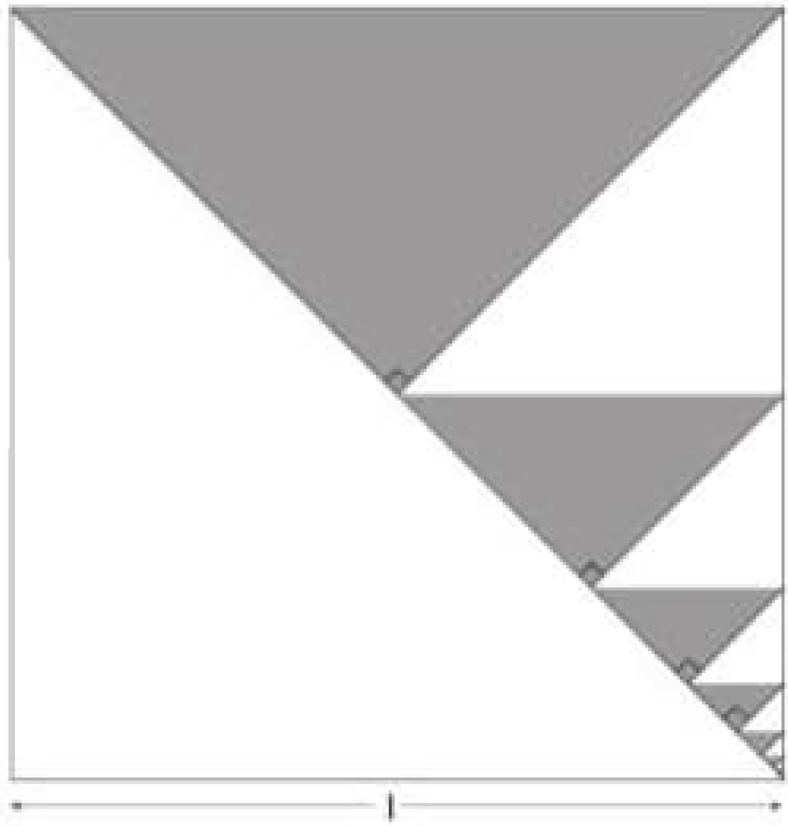
Let $a_n = \dfrac{16}{2^n}$ and $S_n = \displaystyle\sum_{k=0}^{n} a_k$ for $n \geq 0$. Find S_4.

Answer:
(one character per box)

Please print your name in this box

Practice Problem Number 53.

Find the infinite sum of the shaded areas.



Answer:

(one character per box)

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Practice Problem Number 54.

Consider the options

(A) **series is absolutely convergent**

(B) **series is conditionally convergent**

(C) **series is conditionally divergent**

(D) **series is divergent.**

Now put in the first box below the letter that describes the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n},$$

put in the middle box below the letter that describes the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\pi^n},$$

and put in the last box below the letter that describes the series

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}.$$

Answer:
(one character per box)

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Practice Problem Number 55.

Determine whether $\sum_{n=2}^{\infty} \frac{n^2 - 1}{n^2 + 1}$ is convergent or divergent.

Answer:

(put C for convergent or D for divergent)

Please print your name in this box

Please print your name in this box	

Practice Problem Number 56.

Use the integral test to determine whether $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$ is convergent or divergent.

Answer:
(put C for convergent or D for divergent)

Please print your name in this box	

Practice Problem Number 57.

Determine whether the series $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{n \ln(n)}$ **is convergent or divergent.**

Answer:
(put C for convergent or D for divergent)

Please print your name in this box	

Practice Problem Number 58.

Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n-1}$ **is convergent or divergent.**

Answer: $\square$

(put C for convergent or D for divergent)

Please print your name in this box

Practice Problem Number 59.

Let $f(x) = (2 - x)e^x$. Find the Taylor polynomial $p_2(x)$ of order 2 of f centered at 0.

Answer:
(one character per box)

Please print your name in this box	

Practice Problem Number 60.

Find the radius of convergence of $\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n x^n$.

Answer:

(one character per box)

$\square$

Please print your name in this box

Practice Problem Number 61.

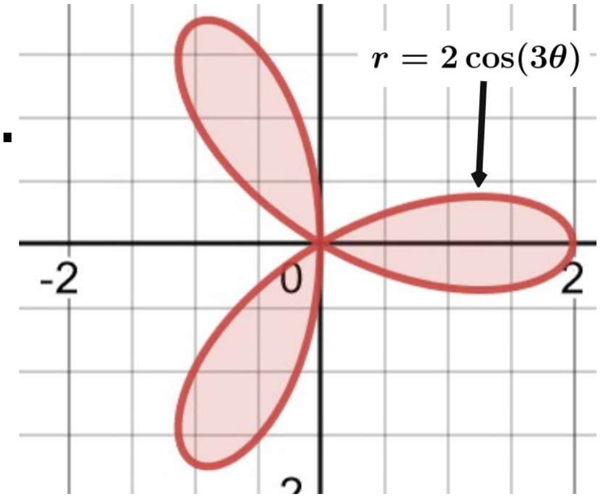
Find the slope of the tangent in the point where the graph of $x = t^3 - t^2$, $y = t^2$ crosses the positive y-axis.

Answer:
(one character per box)

Please print your name in this box

Practice Problem Number 62.

Calculate the area of the shaded region.



Answer:

(one character per box)

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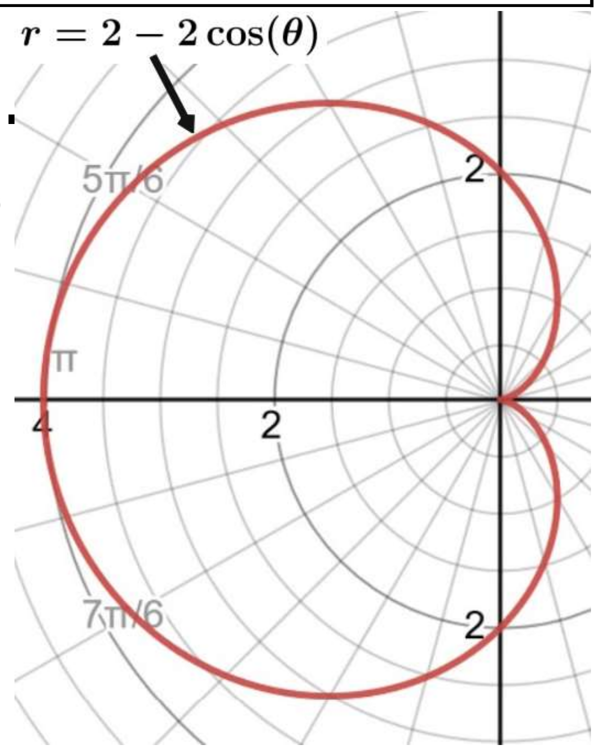
Practice Problem Number 63.

Calculate the length of the given arc.



Answer:
(one character per box)