1. Rewrite

$$u + 2v + 3w = 7, \quad 2u + 5v + 6w = 1, \quad 3u + 6v + 7w = 1$$

as an equation $Ax = b$, find the LDU Decomposition of A, find c such that $Lc = b$, and find x such that $DUx = c$. Give the solution of the original problem and check your solution.

2. Given are the two matrices

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}.$$

Find (if existent) A^T, B^T, AB, BA, AA^T, $I - A$, $2B$, A^{-1}, B^{-1}.

3. Is the set of vectors in $\mathbb{R}^3$ that have zero as the third component a subspace of $\mathbb{R}^3$? How about the set of vectors in $\mathbb{R}^3$ that have a nonnegative number as the third component? (Prove your claims.)

4. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 7 & 8 \\ 3 & 6 & 14 & 12 \end{bmatrix}$. Find the four fundamental subspaces of A and present bases for them as well as their dimensions.