5. We call a matrix P idempotent if $P^2 = P$.
   (a) Give five explicit examples of idempotent 2×2-matrices.
   (b) Find all idempotent 2×2-matrices.
   (c) Let P be idempotent. Prove that $I - P$ is also idempotent.
   (d) Let P be idempotent. Prove the formula $\mathcal{R}(I - P) = \mathcal{N}(P)$.
   (e) Let P be idempotent. Prove the formula $\mathcal{N}(I - P) = \mathcal{R}(P)$.
   (f) Let P be idempotent. Find $\mathcal{N}(P) + \mathcal{R}(P)$.