

1. Solve the system $2u + v = 8$, $4u - \frac{3}{2}v = 9$ using each of the four methods presented in class.
2. Find a polynomial of degree two whose graph goes through the points:
 - (a) $(1, -1)$, $(2, 3)$, and $(3, 13)$;
 - (b) $(1, s_1)$, $(2, s_2)$, and $(3, s_3)$, where $s_1, s_2, s_3 \in \mathbb{R}$.
3. For the following systems of equations, do the following: Rewrite the systems as an equation $Ax = b$, do Gaussian Elimination and write down the elementary matrices needed, find the LDU Decomposition of A , find c such that $Lc = b$ and finally find x such that $DUx = c$:
 - (a) $2u + 4v = 3$, $3u + 7v = 2$;
 - (b) $3u + 5v + 3w = 25$, $7u + 9v + 19w = 65$, $-4u + 5v + 11w = 5$;
 - (c) $u + 2v + 3w = 39$, $u + 3v + 2w = 34$, $3u + 2v + w = 26$;
 - (d) $u + 3v + 5w = 1$, $3u + 12v + 18w = 1$, $5u + 18v + 30w = 1$;
 - (e) $\alpha u + \beta v = 1$, $\beta u + \gamma v = 1$ (where $\alpha, \beta, \gamma \in \mathbb{R}$, $\alpha(\alpha\gamma - \beta^2) \neq 0$).
4. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$.
 - (a) Compute AC , CB , ACB , A^2 , B^2 , CC^T .
 - (b) Find A^n and B^n for all $n \in \mathbb{N}$ (prove your claim using the Principle of Mathematical Induction).
 - (c) Show that A is not invertible. Also show that B is invertible and find B^{-1} .
5. Prove Proposition 1.1(b), i.e., matrix operations are distributive.
6. Use the Gauss-Jordan method to find the inverses of:
 - (a) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$;
 - (b) $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$;
 - (c) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$.