7. Find examples of 2×2-matrices with:
   (a) $A^2 = -I$ (A having only real entries);
   (b) $B^2 = 0$ (but $B \neq 0$);
   (c) $CD = -DC$ (but $CD \neq 0$);
   (d) $EF = 0$ (neither E nor F having any zero entries);
   (e) $AB = AC$ but $B \neq C$;
   (f) $A + B$ is not invertible but A and B are;
   (g) $A + B$ is invertible but A and B are not;
   (h) A and B are symmetric but AB is not.
8. Let A be any matrix. Show that AA^T and A^TA are both symmetric.
9. A real 2×2-matrix is called *symplectic* if $A^T\mathcal{J}A = \mathcal{J}$, where $\mathcal{J} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Characterize symplectic matrices in terms of their entries.
10. If A, B, and $A + B$ are invertible, show that $A^{-1} + B^{-1}$ is invertible and find a formula for its inverse in terms of A, B, $A + B$ and their inverses.
11. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$. Find all matrices M for which $AM = A$.
12. Prove that diagonal matrices of the same order commute.
13. Let D be an arbitrary diagonal matrix. When is D invertible? If it is invertible, what is D^{-1}?
14. Let A and D be square matrices of the same size. Assume that D is diagonal. Describe how AD looks like. How about DA?
15. Let A be a matrix of size $m \times n$. Find a matrix P such that P multiplied with A exchanges the ith row and the jth row of A. What needs to be done if the ith column and the jth column of A should be exchanged?
16. Suppose that $(I + A)^{-1}A = B$ holds for two matrices A and B.
    (a) Prove that A and B commute.
    (b) Prove that, if B is invertible and diagonal, then also A is invertible and diagonal.
17. Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. Find all vectors v that satisfy $Av = 0$.
18. Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 4 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 0 \\ 5 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 6 \\ 10 \\ 14 \end{bmatrix}$. Find numbers a, b, and c with $av_1 + bv_2 + cv_3 = 0$.