66. Find all eigenvalues and corresponding eigenvectors for the following matrices A. Also find the trace and the determinant of the matrices. Then verify that the trace is the sum and the determinant is the product of the eigenvalues.

(a) $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$;

(b) $A = \begin{bmatrix} -3 & 1 \\ -7 & 5 \end{bmatrix}$;

(c) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$;

(d) $A = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$;

(e) $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$;

(f) $A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$;

(g) $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$;

(h) The two matrices in problem 5.1.14.

67. For the invertible matrices of the previous problem, do the following: Calculate the inverse and its eigenvalues. Guess a connection between the eigenvalues of an invertible A and the eigenvalues of A^{-1} and prove it.

68. For three matrices of your choice from above, do the following: Write down the characteristic polynomial. Then plug A instead of λ, i.e., for λ^2 use A^2 and so on. Compute the result in all three cases. Guess what the result in general is and try to prove it.

69. For three matrices of your choice from above, do the following: Find the matrix B given by $B = 2A^2 + A - 3I$ and calculate the eigenvalues of B. Guess what the result in general is and prove it.

70. Work again on Problems 1–69 to be prepared for the final examination.