29. Calculate the Laplace transforms of the functions given in rows 3, 5, 7, 9, 10, 11 of Table 6.2.1 in the textbook.

30. Use the definition to find the Laplace transforms of the following functions:

$$
\text{(a)} \quad f(t) = \begin{cases} -1 & 0 \le t < 1 \\ 1 & t \ge 1 \end{cases}
\qquad\qquad
\text{(b)} \quad f(t) = \begin{cases} t & 0 \le t < 1 \\ 1 & t \ge 1 \end{cases}
$$

$$
\text{(c)} \quad f(t) = \begin{cases} \sin t & 0 \le t < \pi \\ 0 & t \ge \pi \end{cases}
\qquad\qquad
\text{(d)} \quad f(t) = \begin{cases} t & t \ge 1 \\ 0 & 0 \le t < 1. \end{cases}
$$

31. Use the Laplace transform to solve the following IVPs:

   (a) $x' - 5x = 0$, $x(0) = 4$;

   (b) $x' - x = 1$, $x(0) = 0$;

   (c) $x' + 4x = e^{-4t}$, $x(0) = 2$;

   (d) $x'' + 5x' + 6x = 0$, $x(0) = 1$, $x'(0) = -5$;

   (e) $x'' + 5x' + 4x = 0$, $x(0) = 1$, $x'(0) = 0$;

   (f) $2x''' + 3x'' - 3x' - 2x = e^{-t}$, $x(0) = 0$, $x'(0) = 0$, $x''(0) = 1$;

   (g) $x'''' - x = 0$, $x(0) = 1$, $x'(0) = 0$, $x''(0) = -1$, $x'''(0) = 0$.

32. Find out what the gamma function is and state and prove some of the properties of this function.