

6.[14] Define the function f by the formula

$$f(t,y) = \begin{cases} (t-y)^2 & \text{if } t \leq y, \\ (t-y)^{3/2} & \text{if } t > y. \end{cases}$$

~~$\frac{\partial f}{\partial t}$~~ or $\frac{\partial f}{\partial y}$ inside of a Wronskian computation does not count
 +6 if $\frac{\partial f}{\partial t}$ (instead of $\frac{\partial f}{\partial y}$)

For what values of t_0 and y_0 does the initial value problem

$$y' = f(t,y), \quad y(t_0) = y_0.$$

have a unique solution defined on some open interval $t_0 - h < t < t_0 + h$ containing t_0 ?

8 [If $f(t,y)$ and $\frac{\partial f}{\partial y}(t,y)$ are cont. in a neighborhood containing (t_0, y_0) , then there is a unique soln to the IVP on some interval containing t_0 .

+7 if anything about continuity/discontinuity/ $\frac{\partial f}{\partial y}$

2 [First, $f(t,y)$ is cont. for all (t,y) since when $t=y$, $f=0$ (both "pieces" of the piecewise func agree). y' or $f(t,y)$ defined everywhere

if no cont. check at $t=y$, then $+1/2$ (for either)

or if "defined" instead of continuous

$\frac{\partial f}{\partial t}$ instead of $+1/2$

Next,
$$\frac{\partial f}{\partial y}(t,y) = \begin{cases} -2(t-y) & \text{if } t \leq y \\ -\frac{3}{2}(t-y)^{1/2} & \text{if } t > y \end{cases}$$

note: the square root is well defined since $t-y > 0$

is also cont. for all (t,y) since when $t=y$, $\frac{\partial f}{\partial y}(t,y) = 0$ (again, both "pieces" agree).

2 [\Rightarrow there is a unique soln for all (t_0, y_0) .

must earn all other points in order to earn these 2 points (unless they check $\frac{\partial f}{\partial t}$ instead of $\frac{\partial f}{\partial y}$)