Mathematics 204

Fall 2012

Exam I

[1] Your Printed Name: _____

[1] Your Instructor's Name: _____

Your Section (or Class Meeting Days and Time): _____

1. **Do not open this exam until you are instructed to begin.**

2. All cell phones and other electronic noisemaking devices must be **turned off or completely silenced** (i.e. not on vibrate) for the duration of the exam.

3. You are **not allowed to use a calculator** on this exam.

4. Exam I consists of this cover page and 6 pages of problems containing 6 numbered problems.

5. Once the exam begins, you will have 60 minutes to complete your solutions.

6. **Show all relevant work. No credit** will be awarded for unsupported answers and partial credit depends upon the work you show.

7. You may use the back of any page for extra scratch paper, but if you would like it to be graded, clearly indicate in the space of the original problem where the work is to be found.

8. The symbol [17] at the beginning of a problem indicates the point value of that problem is 17. The maximum possible score on this exam is 100.

	0	1	2	3	4	5	6	Sum
points earned								
maximum points	2	15	17	17	17	18	14	100

1.[15] Determine all values of r for which the differential equation $4t^2 y'' + 12ty' + 3y = 0$ has solutions of the form $y = t^r$ for $t > 0$.

2.[17] Solve the initial value problem $4t^2 - ty' = 2y$, $y(1) = 2$.

3.[17] Solve the initial value problem $e^t y' - \left(\dfrac{1+e^t}{y+1} \right) y = 0, \quad y(0) = 1.$

4.[17] A 1000 gallon tank originally holds 300 gallons of water solution containing 100 pounds of salt. Then water containing 2 pounds of salt per gallon is poured into the tank at a rate of 5 gallons per minute, and the well-stirred mixture is allowed to leave the tank at a rate of 3 gallons per minute.

(a) How long will it take before the tank begins to overflow?

(b) Set up, **BUT DO NOT SOLVE,** an initial value problem that models the amount of salt in the tank at all times prior to the moment when the tank overflows.

5.[18] Find the general solutions to the following differential equations.

(a) $y'' - 3y' = 0$

(b) $y'' - 2y' + 5y = 0$

(c) $y'' + 6y' + 9y = 0$

6.[14] Define the function f by the formula

$$f(t,y) = \begin{cases} (t-y)^2 & \text{if } t \le y, \\ (t-y)^{3/2} & \text{if } t > y. \end{cases}$$

For what values of t_0 and y_0 does the initial value problem

$$y' = f(t,y), \quad y(t_0) = y_0,$$

have a unique solution defined on some open interval $t_0 - h < t < t_0 + h$ containing t_0?