Mathematics 204

Fall 2012

Exam III

Your Printed Name: ______________________________________

Your Instructor's Name: ___________________________________

Your Section (or Class Meeting Days and Time): ____________________________

1. **Do not open this exam until you are instructed to begin.**
2. All cell phones and other electronic devices must be **turned off or completely silenced** (i.e. not on vibrate) for the duration of the exam.
3. You are **not allowed to use a calculator** on this exam.
4. Exam III consists of this cover page, 5 pages of problems containing 5 numbered problems, and a short table of Laplace transforms.
5. Once the exam begins, you will have 60 minutes to complete your solutions.
6. **Show all relevant work. No credit** will be awarded for unsupported answers and partial credit depends upon the work you show.
7. Express all solutions in real-valued, simplified form.
8. You may use the back of any page for extra scratch paper, but if you would like it to be graded, clearly indicate in the space of the original problem where the work is to be found.
9. The symbol [22] at the beginning of a problem indicates the point value of that problem is 22. The maximum possible score on this exam is 100.

	1	2	3	4	5	**Sum**
points earned						
maximum points	22	20	14	22	22	100

1.(a) [18] Find the solution $y = y(t)$ of the initial value problem $y'' + 3y' + 2y = \delta(t-5)$, $y(0) = 0$, $y'(0) = 1$.

(b) [4] Which is greater, $y(6)$ or $y(1)$? Justify your answer.

2.[20] Consider two interconnected tanks. Tank 1 initially contains 40 grams of sugar dissolved in 60 liters of water and Tank 2 initially contains 50 liters of water with 25 grams of sugar. Water containing 2 grams of sugar per liter flows into Tank 2 at a rate of 3 liters per minute. The well-stirred mixture drains from Tank 2 at a rate of 4 liters per minute, of which some flows into Tank 1 at a rate of 1.5 liters per minute while the remainder leaves the system. The well-stirred mixture in Tank 1 flows back into Tank 2 at a rate of 1.5 liters per minute. If $S_1(t)$ and $S_2(t)$ denote the amounts of sugar at time t in Tanks 1 and 2, respectively, set up, **BUT DO NOT SOLVE,** an initial value problem that models the flow process.

3.[14] If $\mathbf{A}(t)=\begin{pmatrix} 2e^{2t} & e^{-t} \\ 2e^{t} & 3e^{-2t} \end{pmatrix}$, find $\frac{d}{dt}\left(\mathbf{A}^{-1}(t)\right)$.

4.[22] Solve the integral equation $y(t) = 3t^2 - e^{-t} - \int_0^t y(u)e^{t-u}\,du$.

5.(a) [20] Solve the initial value problem $\mathbf{x}' = \begin{pmatrix} -3 & -1 \\ 2 & -1 \end{pmatrix}\mathbf{x}$, $\mathbf{x}(0) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$.

(b) [2] Describe the behavior of the solution as $t \to \infty$.

SHORT TABLE OF LAPLACE TRANSFORMS

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
1. e^{at}	$\dfrac{1}{s-a}$
2. t^n	$\dfrac{n!}{s^{n+1}}, \quad n = 0,1,2,3....$
3. $\sin(bt)$	$\dfrac{b}{s^2+b^2}$
4. $\cos(bt)$	$\dfrac{s}{s^2+b^2}$
5. $f * g(t)$	$F(s)G(s)$
6. $f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - ... - f^{(n-1)}(0)$
7. $e^{ct} f(t)$	$F(s-c)$
8. $u_c(t) f(t-c)$	$e^{-cs} F(s)$
9. $\delta(t-c)$	e^{-cs}