

**Part A:** Fill in only the boxes and do your work on a separate sheet.

1. Determine how many strings can be formed by ordering the letters ABCDE subject to the conditions given (enter the number values):

(a) ACE must be together in any order:  ;

(b) ACE is contained as a substring:  ;

(c) either the substring  $EA$  or the substring  $AE$  is contained:  ;

(d) contains neither of the substrings  $AB$  and  $CD$ :  ;

(e) contains neither of the substrings  $AB$  and  $BE$ :  .

2. This exercise refers to a club consisting of six distinct men and seven distinct women. In how many ways can we select a committee consisting of (enter the number values)

(a) three men and four women?  ;

(b) five persons?  ;

(c) four persons that has at most one man?  ;

(d) four persons that has persons of both sexes?  ;

(e) four persons that has at least one woman?  .

**Part B:** For the remaining problems, show your work clearly, explaining each step of the proofs. Use only the space allocated for each problem (use separate sheets of paper for additional work).

3. Use “integrating factors” to find the solution of  $y_{n+1} = 1 - 2y_n$ ,  $y_0 = 0$ .

4. Suppose we have \$10000 in a savings account that pays an annual interest rate of 3% and now start taking at the beginning of each year \$500. Let  $y_n$  be the money in the account at the end of the  $n$ th year. Find a recurrence relation for  $y$  and solve it. Will the account ever be empty? If so, when?

5. Find the solution of  $y_{n+2} = y_{n+1} + y_n$ ,  $y_0 = 1$ ,  $y_1 = 3$ .

Use the following two sheets to do scratchwork for Part A and any additional work for Part B. Note: The contents on these sheets will **not** be graded.