Let $P(n)$ be the propositional function

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

We show that

(1) $$\boxed{\forall n \in \mathbb{N} \quad P(n)}$$

is true.

## 1. First Step

Since $1 = \frac{1 \cdot 2}{2}$, we find that

(2) $$\boxed{P(1)}$$

is true.

## 2. Second Step

Let $n \in \mathbb{N}$. If $P(n)$ is false, then $P(n) \to P(n+1)$ is true. Now assume that $P(n)$ is true. Then

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

Hence

$$\begin{aligned} \sum_{k=1}^{n+1} k &= \sum_{k=1}^{n} k + (n+1) \\ &= \frac{n(n+1)}{2} + (n+1) \\ &= \frac{n(n+1) + 2(n+1)}{2} \\ &= \frac{(n+2)(n+1)}{2} \end{aligned}$$

so that $P(n+1)$ is true, and now $P(n) \to P(n+1)$ is true. Altogether, we now find that

(3) $$\boxed{\forall n \in \mathbb{N} \quad P(n) \to P(n+1)}$$

is true.

## 3. Conclusion

Since the statements (2) and (3) are both true, we use the *Principle of Mathematical Induction* to conclude that (1) is true.