

1. Define the total variation of a function  $f$  on an interval  $[a, b]$ .
2. What does the symbol  $BV[a, b]$  mean?
3. Suppose  $f$  is nondecreasing on  $[a, b]$ . Find  $V_a^b f$ .
4. Every  $f \in BV[a, b]$  can be written as the difference of two nondecreasing functions. How can these two functions be chosen?
5. When is a function  $f : [a, b] \rightarrow \mathbb{R}$  called “Riemann-Stieltjes integrable”?
6. State the “Fundamental Inequality for Riemann-Stieltjes Integrals”.
7. If  $f$  is continuous and  $g$  is 0 up to  $t$  but makes a jump to  $p$  at  $t$  and stays at  $p$  afterwards, find the Riemann-Stieltjes integral of  $f$  with respect to  $g$ .
8. State the “Integration by Parts Formula” for Riemann-Stieltjes integrals.
9. State the “Main Existence Theorem” for Riemann-Stieltjes integrals.
10. For which kind of integrands and integrators did we define lower and upper sums? How are they defined? What are the lower and upper Riemann-Stieltjes integrals? What is the “Main Existence Theorem” in this situation?
11. Define “pointwise” and “uniform” convergence of a sequence of functions  $\{f_n\}$  to a function  $f$  on a set  $E$ .
12. Show that  $\{x^n\}$  converges uniformly on  $[0, 1/2]$  but not on  $[0, 1]$ .
13. State the “Cauchy Criterion” for uniform convergence.
14. Describe the “Weierstraß  $M$ -Test”.
15. State the three main theorems on continuity, integrability, and differentiability of the limit function. Also state the corresponding corollaries for series.
16. What is a norm? What is a metric? What is a Banach space?
17. Which norm makes  $C[a, b]$  into a Banach space?
18. What is an equicontinuous family of functions?
19. State the “Arzelà-Ascoli Theorem”.
20. State the “Weierstraß Approximation Theorem”.