48. Evaluate $\sum_{k=1}^{\infty} 1/(k2^k)$ by integrating the geometric series.
49. Evaluate $\sum_{k=1}^{\infty} k/2^k$ by differentiating the geometric series.
50. Show that f has derivatives of all orders at $x = 0$ but is not analytic:
$$f(x) = \begin{cases} \exp(-1/x^2) & \text{if} \quad x \neq 0 \\ 0 & \text{if} \quad x = 0. \end{cases}$$
51. Use Taylor expansion of $\log(1+x)$ to find $\sum_{n=0}^{\infty}(-1)^n/(n+1)$.
52. Use Taylor expansion of $\arctan(x)$ to find $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots$.
53. Prove the following Tauberian Theorem: Suppose $f(x) = \sum_{n=0}^{\infty} a_n x^n$ converges for $|x| < 1$ and $\lim_{x\to 1} f(x) = s$. If $na_n \to 0$ as $n \to \infty$, then $\sum_{n=0}^{\infty} a_n$ converges and is equal to s.
54. Show that $E(x) \geq 1 + x$ for all $x \in \mathbb{R}$.
55. Work on problems 4, 5, and 6 of Chapter 8 in the textbook.