62. For the following functions, calculate the integral between 0 and 1 once by the Riemann method and another time by the Lebesgue method (as illustrated in Example 4.1 in class):

    (a) $f(x) = x$;

    (b) $f(x) = \sqrt{x}$.

63. Prove Lemma 4.3 from the lecture notes for $N = 1$ and $N = 2$.

64. Show that each countable union can be written as a countable union of pairwise disjoint sets.

65. Prove that each open set in $\mathbb{R}^N$ is the countable union of closed intervals which have pairwise disjoint interiors.

66. Let $\{A_n\}$ be a sequence of sets. The set of elements that are in almost all sets A_n is denoted by $\liminf_{n\to\infty} A_n$. The set of elements that are in inifinitely many A_n is denoted by $\limsup_{n\to\infty} A_n$. If these limits are the same, this set is denoted by $\lim_{n\to\infty} A_n$. Show:

    (a) $\liminf_{n\to\infty} A_n \subset \limsup_{n\to\infty} A_n$;

    (b) $\liminf_{n\to\infty} A_n = \cup_{n=1}^{\infty} \cap_{k=n}^{\infty} A_k$;

    (c) $\limsup_{n\to\infty} A_n = \cap_{n=1}^{\infty} \cup_{k=n}^{\infty} A_k$;

    (d) $(\limsup_{n\to\infty} A_n)^C = \liminf_{n\to\infty} A_n^C$;

    (e) $(\liminf_{n\to\infty} A_n)^C = \limsup_{n\to\infty} A_n^C$;

    (f) $\lim_{n\to\infty} A_n = \cup_{n=1}^{\infty} A_n$ if $A_k \subset A_{k+1}$ for all $k \in \mathbb{N}$;

    (g) $\lim_{n\to\infty} A_n = \cap_{n=1}^{\infty} A_n$ if $A_k \supset A_{k+1}$ for all $k \in \mathbb{N}$.

67. Calculate the limits from the previous problem if $A_{2n} = [0, 1/2]$ and $A_{2n-1} = [0, 1]$ for $n \in \mathbb{N}$.

68. For nonempty sets A and B the expression $\text{card}A = \text{card}B$ $(<)$ means that there is a function $f : A \to B$ which is bijective (injective but not bijective). Show that $\text{card}\mathcal{P}(X) > \text{card}X$ for each set $X \neq \emptyset$.