77. Check whether the following functions are harmonic:
   (a) $u(x,y) = x^3 - 3x^2y$ in $\mathbb{R}^2$;
   (b) $u(x,y) = e^x \cos y$ in $\mathbb{R}^2$;
   (c) $u(x,y,z) = 3x^2 - y^2 - z^2$ in $\mathbb{R}^3$;
   (d) $u(x,y,z) = e^x \sin \frac{y}{\sqrt{2}} \cos \frac{z}{\sqrt{2}}$ in $\mathbb{R}^3$;
   (e) $f(x) = \frac{1}{\|x\|^{n-2}}$ in $\mathbb{R}^n \setminus \{0\}$.
78. Determine the points where the Cauchy–Riemann equations are satisfied for
   (a) $f(z) = e^z$;
   (b) $f(z) = \bar{z}$;
   (c) $f(z) = z^2 e^z$;
   (d) $f(z) = |z|^2$.
79. Determine v such that $f = u + \mathrm{i}v$ with $u(x + \mathrm{i}y) = 2x^3y - 2xy^3 + x^2 - y^2$ and $f(0) = \mathrm{i}$ satisfies the Cauchy-Riemann equations.
80. Let $D = \left\{ \binom{x}{y} : x^2 + y^2 < 9 \right\}$. Find the maximum value of u in $\overline{D}$, where u solves

$$u_{xx} + u_{yy} = 0 \quad \text{in} \quad D, \qquad u = \cos\left(\frac{\theta}{2}\right) \quad \text{on} \quad \partial D.$$

81. Solve $\Delta u = 0$, $0 < x < a$, $0 < y < b$, $u(x,0) = 0$, $u(x,b) = g(x)$, $u(0,y) = 0$, $u(a,y) = 0$.
82. Solve $\Delta u = 0$ on $D = \left\{ \binom{x}{y} : x^2 + y^2 < a^2 \right\}$, $u = a^k \cos(k\theta)$ on ∂D.
83. Solve $\Delta u = 0$ on $D = \left\{ \binom{x}{y} : x^2 + y^2 < 4 \right\}$, $u = 32\sin(5\theta)$ on ∂D.
84. Find the transformed Laplacian operator in three dimensions when spherical coordinates are introduced.