54. Use Fourier series to solve the following boundary value problems:

    (a) $u_t = u_{xx}$ $(0 < x < 1,\ t > 0)$, $u(0,t) = u(1,t) = 0$, $u(x,0) = x$.

    (b) $u_{tt} = u_{xx}$ $(0 < x < \pi,\ t > 0)$, $u(0,t) = u(\pi,t) = 0$, $u(x,0) = 0$, $u_t(x,0) = x^2(\pi - x)^2$.

    (c) $u_t = u_{xx}$ $(0 < x < \pi,\ t > 0)$, $u(0,t) = u(\pi,t) = 0$, $u(x,0) = x(\pi - x)$.

    (d) $u_t = u_{xx}$, $(0 < x < \pi,\ t > 0)$, $u_x(0,t) = u_x(\pi,t) = 0$, $u(x,0) = \cos^4 x$.

    (e) $9u_{tt} = u_{xx}$ $(0 < x < \pi,\ t > 0)$, $u(0,t) = u(\pi,t) = 0$, $u(x,0) = 0$, $u_t(x,0) = x(x - \pi)$.

55. Find the Fourier sine series in $(0,\pi)$ of $f(x) = \cos x$.

56. Find the Fourier cosine series in $(0,\pi)$ of $f(x) = \cos^3 x$.

57. Find the Fourier coefficients of f on $[-l,l]$ if f is

    (a) even;

    (b) odd.

58. Find the Fourier coefficients of f on $[-\pi,\pi]$ for

    (a) $f(x) = x$;

    (b) $f(x) = |x|$;

    (c) $f(x) = |\sin x|$;

    (d) $f(x) = x^2$;

    (e) $f(x) = \cosh(\alpha x)$, $\alpha \neq 0$;

    (f) $f(x) = -3$ if $-\pi \le x < 0$, $f(x) = 0$ if $x = 0$, and $f(x) = 1$ if $0 < x \le \pi$.

59. Use the previous problem to find the following infinite series:

    (a) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \ldots$;

    (b) $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{9^2} + \ldots$;

    (c) $\frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \frac{1}{7\cdot 9} + \ldots$;

    (d) $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \ldots$;

    (e) $\frac{1}{\alpha} + \sum\limits_{n=1}^{\infty} \frac{2\alpha}{\alpha^2+n^2}$.

60. Use the previous problem to determine the value of $\sum\limits_{n=1}^{\infty} \frac{1}{n^2}$.

61. Find the complex form of the Fourier series of $f(x) = e^x$.