# Section 2.2

Separable Equations

## A Note about Section 2.1

Astute observers will note that we are not covering Section 2.1.

Section 2.1 has no associated homework problems.

Section 2.1 does provide some motivation for why we might want to solve "simple" first-order equations using a discussion of Newton's Second Law

$$F = m\frac{d^2y}{dx^2}$$

If you're interested, read it after we have finished section 2.2.

## First-Order Equations

A first-order ODE is an ODE which can be written in the form

$$\frac{dy}{dt} = f(t, y)$$

A first-order linear ODE is an ODE which can be written in the form

$$a_1(t)y'(t) + a_0(t)y(t) = f(t)$$

A first-order separable ODE is an ODE which can be written in the form

$$\frac{dy}{dt} = g(t)h(y)$$

## Solving Separable Equations

To solve an ODE of the form

$$\frac{dy}{dt} = g(t)h(y)$$

begin by rewriting as

$$\frac{dy}{h(y)} = g(t)dt$$

Then, integrate both sides to solve.

## Example 1

Solve the IVP

$$y' = \frac{3x^2}{3y^2 - 4}, y(1) = 0$$

and determine the largest interval in which the solution is valid.

## Example 2

Find the general solution to the ODE

$$y'(t) = y^2 - 4$$

## Example 3

Solve the DE

$$\csc y + \sec^2 x \frac{dy}{dx} = 0$$