Section 6.4

Nonhomogeneous Linear Equations:
Variation of Parameters

Variation of Parameters

Consider the standard form differential equation

$$y^{(n)} + a_{n-1}(t)y^{(n-1)} + \cdots + a_1(t)y' + a_0(t)y = g(t)$$

If $\{y_1, y_2, \ldots, y_n\}$ is a fundamental set of solutions of the associated homogeneous equation, then a particular solution of the equation is

$$y_p(t) = u_1(t)y_1(t) + u_2(t)y_2(t) + \cdots + u_n(t)y_n(t)$$

If $W(t)$ is the Wronskian of $\{y_1, y_2, \ldots, y_n\}$ and $W_i(t)$ is the determinant obtained by replacing the i^{th} column of W with the column $\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$, then

$$u_i(t) = \int \frac{g(t)W_i(t)}{W}\,dt$$

Example 1

Find the general solution of

$$y''' + y' = \sec t$$