Chapter 5

Forward and Futures Prices

74

Definition 5.1

- An investment asset is an asset which is held by a significant number of people for investment purposes
- A consumption asset is held primarily for consumption purposes

75

Remark 5.2

Short selling involves selling an asset that is not owned. The broker borrows the asset from another client and sells it. At some stage the asset must be replaced. Until then, dividends or other benefits must be paid to the owner and a margin account must be maintained.

76

Theorem 5.3

For an investment asset providing no income, we have

$$F_0=F(0,T)=S_0e^{rT}$$

and

$$F(t,T)=S(t)e^{r(T-t)}.$$

77

Example 5.4

Consider a long forward contract to purchase a non-dividend-paying stock in 3 months. Assume S_0=40 and r=5% cc. Find F_0.

78

Example 5.5

Suppose the stock price on 1 April 2023 is 10% lower than that on 1 January 2023 and r=6%. What is the percentage drop of the forward price on 1 April 2023 compared to that on 1 January 2023 for a forward with delivery on 1 October 2023?

79

Theorem 5.6

For an investment asset providing income with present value I, we have

$F_0=(S_0-I)e^{rT}$.

80

Example 5.7

Consider a 10-month forward on a stock with $S_0=50$, $r=8\%$, that pays dividends of \$0.75 after 3, 6, and 9 months. Find F_0.

81

Example 5.8

Consider a long 9-month forward to purchase a coupon-bearing bond with current price \$900. The coupon payment is \$40 after 4 months. Suppose the 4-month/9-month risk-free rates are 3%/4%. Find F_0.

82

Theorem 5.9

For an investment asset providing income continuously at a known yield q (cc), we have

$F_0=S_0e^{(r-q)T}$.

83

Example 5.10

Consider a forward on a stock paying a continuous dividend at rate 3%, $T=1/2$ year, $S_0=98.34$, $r=10\%$. Find F_0.

84

Theorem 5.11

The value of a long forward contract at time t is

$V(t)=[F(t,T)-F(0,T)]e^{-r(T-t)}$.

85

Corollary 5.12

The value of a long forward on an investment asset providing

- no income is $S(t)-F_0e^{-r(T-t)}$
- known income with present value I is $S(t)-I(t)-F_0e^{-r(T-t)}$
- known yield at rate q is $S(t)e^{-q(T-t)}-F_0e^{-r(T-t)}$.

86

Example 5.13

A long forward contract on a non-dividend-paying stock was entered into some time ago. It currently has 6 months to maturity. The stock price is \$25, the delivery price \$24, r=0.1. What is its value now?

87

Remark 5.14

A stock index can be viewed as an investment asset paying a dividend yield. Thus, by Theorem 5.9, if q is the average dividend yield on the portfolio represented by the index during the life of a futures contract, then

$$F_0=S_0e^{(r-q)T}$$

(otherwise, index arbitrage occurs).

88

Example 5.15

Consider a 3-month futures contract on the S&P 500. Suppose that the stocks underlying the index provide a dividend yield of 1% pa, the current value of the index is 800, and r=6% cc. Find F_0.

89

Remark 5.16

A foreign currency is analogous to a security providing a dividend yield, which is equal to the foreign interest rate r_f. Thus, if S_0 (in \$) is the spot exchange rate and r is the US interest rate, then the forward or futures price on a foreign currency is

$$F_0=S_0e^{(r-r_f)T}$$

(this is also known as the interest rate parity relationship from International Finance).

90

Example 5.17

Suppose 2-year interest rates in Australia and the US are 5% and 7%, respectively, cc. Assume S_0=0.62 USD per AUD. Find F_0.

91

Remark 5.18

Storage costs can be treated as negative income with present value U or negative yield u. Hence, for commodities that are investment assets, we have

$F_0=(S_0+U)e^{rT}$ or $F_0=S_0e^{(r+u)T}$.

Example 5.19

Consider a 1-year futures on an investment asset providing no income, but it costs $2 (at the end of the year) to store the asset. Assume S_0=450 and r=7% cc. Find F_0.