

Missouri S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

**Chapter 6**

**Interest Rate Futures**

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**Definition 6.1**

Daycount conventions in the US are given as  $X/Y$ , where  $X$  and  $Y$  define how to count the number of days between two dates and in the reference period, respectively, and are

- actual/actual (e.g., for US Treasury bonds)
- 30/360 (e.g., for US corporate and municipal bonds)
- actual/360 (e.g., for US money market instruments).

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**Example 6.2**

(a) For a Treasury bond with  $F=100$ , coupon rate 8% with coupons paid on March 1 and September 1, 2023, find the interest earned between March 1 and July 3.

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**Example 6.2 (continued)**

(b) For a corporate bond with the same data as in (a), answer the same question.

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**Definition 6.3**

- Treasury bonds ( $F=100$ ) and futures on them are **quoted** as  $x-y$ , which means that the **quoted price** equals  $\$(x+y/32)$ .
- The **cash price** (or **dirty price**) equals to the quoted price (or **clean price**) plus the accrued interest since the last coupon date.

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**Example 6.4**

Find the cash price of an 11% coupon Treasury bond with  $F=100,000$  maturing on July 10, 2028 with a quote of 95-16 on March 5, 2023.

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### Remark 6.5

We discuss Treasury bond futures traded at CBOT. One contract involves delivery of \$100,000 face value of bonds. Any government bond with more than 15 years to maturity may be delivered by the party with the short futures position. Each bond has a **conversion factor** which is equal to the quoted price the bond would have per dollar of the principal on the first day of the delivery month assuming that interest for all maturities is 6% pa with semi-annual compounding.

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### Remark 6.5 (continued)

All dates are rounded down to the nearest 3 months. If, after rounding, the bond lasts for an integer number of 6-month periods, the first coupon is assumed to be paid in 6 months. If not, the first coupon is assumed to be paid in 3 months and accrued interest is subtracted. Using the CF, cash received for each \$100 face value of bond delivered is  $SP \times CF + AI$ .

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### Example 6.6

If SP is 90-00, CF of the bond delivered is 1.3800, and AI on this bond at delivery is \$3 per \$100, then the party who is with short position in one futures contract would deliver bonds with face value \$100,000 and receives how much?

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### Example 6.7

(a) Find the CF of a 10% coupon bond with 20 years and 2 months to maturity.  
 (b) Find the CF of an 8% coupon bond with 18 years and 4 months to maturity.

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### Definition 6.8

The **cheapest-to-deliver** bond is the one such that  $(QP + AI) - (SP \times CF + AI)$  is least.

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### Example 6.9

Assume the most recent SP is 93-08 and the party with the short position has decided to deliver. Find the CTDB.

Bond	QP (\$)	CF
1	99.50	1.0382
2	143.50	1.5188
3	119.75	1.2615

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**Example 6.10**

Suppose that, in a Treasury bond futures contract, it is known that the CTDB will be a 12% coupon bond with  $CF=1.4000$  and delivery in 270 days,  $r=10\%$  pa cc,  $QP=120$ . The last coupon date was 60 days ago, the next one is in 122 days. Find the quoted price on the futures.