

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

Chapter 9

Properties of Stock Options

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Proposition 9.1

If we let c, p, C, P be the prices of a European call, European put, American call, American put, respectively (all on the same stock, same K and same T), then

$0 \leq c \leq C$ and $0 \leq p \leq P$.

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Theorem 9.2

For a stock paying no dividends,

$(S_0 - Ke^{-rT})^+ \leq c < S_0$.

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Example 9.3

For a European call on a non-dividend paying stock with $S_0=51$, $K=50$, $T=1/2$, $r=0.12$, find the bounds on c .

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Theorem 9.4

For a stock paying no dividends,

$(Ke^{-rT} - S_0)^+ \leq p < Ke^{-rT}$.

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Example 9.5

For a European put on a non-dividend paying stock with $S_0=38$, $K=40$, $T=1/4$, $r=0.1$, find the bounds on p .

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Remark 9.6

- Theorems 9.2 and 9.4 can be graphically summarized
- Note also that by Corollary 5.12 (first part), $S_0 - Ke^{-rT} = V(0)$ is the value of a long forward contract on the non-dividend paying stock

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Theorem 9.7

Put-call parity holds:

$$c - p = S_0 - Ke^{-rT}.$$

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Example 9.8

Suppose a stock paying no dividends trades at \$15.60 a share. European calls on the stock with strike price \$15 and exercise date in 3 months are trading at \$2.83, $r=6.72\%$ (cc). What is the price of a European put on the same stock with the same strike price and the same exercise date?

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Theorem 9.9

For a stock paying no dividends,

$$C = c.$$

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Corollary 9.10

For a stock paying no dividends,

$$(S_0 - Ke^{-rT})^+ \leq C < S_0.$$

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Example 9.11

Suppose $S=10$, strike price of American put expiring in one year is \$80, and $r=0.16$. Here it is better to do early exercise.

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Theorem 9.12

For a stock paying no dividends,
 $(K - S_0)^+ \leq P < K$.

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Theorem 9.13

Put-call parity estimates hold:
 $S_0 - K \leq C - P \leq S_0 - Ke^{-rT}$
 (for a stock paying no dividends).

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Example 9.14

Suppose $S_0=19$, $K=20$, $T=5/12$,
 $r=0.1$, $C=1.50$. Find the bounds on P .

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Theorem 9.15

For a stock paying dividends with present value D ,

- $(S_0 - D - Ke^{-rT})^+ \leq c < S_0 - D$
- $(Ke^{-rT} + D - S_0)^+ \leq p < Ke^{-rT}$
- $c - p = S_0 - D - Ke^{-rT}$

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Theorem 9.15 (continued)

- $S_0 - D - K \leq C - P \leq S_0 - Ke^{-rT}$
- $\max\{0, S_0 - D - Ke^{-rT}, S_0 - K\} \leq C < S_0$
- $\max\{0, Ke^{-rT} + D - S_0, K - S_0\} \leq P < K$

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Remark 9.16

In the rest of this chapter, we discuss the dependence of c , p , C , P on K , S , T , and r (varying only one variable while keeping the others constant; all on stocks paying no dividends).

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Proposition 9.17

c is nonincreasing in K while p is nondecreasing in K, i.e.,
 $c(K) \geq c(K')$ and $p(K) \leq p(K')$
 for $K \leq K'$.

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Theorem 9.18

c and p are Lipschitz continuous in K, namely,
 $|c(K) - c(K')| \leq e^{-rT} |K - K'|$
 and
 $|p(K) - p(K')| \leq e^{-rT} |K - K'|$.

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Theorem 9.19

c and p are convex in K, i.e.,
 $K \leq K'$ and $0 < \alpha < 1$
 implies
 $c(\alpha K + (1 - \alpha)K') \leq \alpha c(K) + (1 - \alpha)c(K')$
 and
 $p(\alpha K + (1 - \alpha)K') \leq \alpha p(K) + (1 - \alpha)p(K')$.

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Theorem 9.20

If $S \leq S'$, then

- $c(S) \leq c(S')$ and $p(S) \geq p(S')$
- $c(S') - c(S) \leq S' - S$
- $p(S) - p(S') \leq S' - S$
- c and p are convex in S.

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Theorem 9.21

For American options, Proposition 9.17 holds, Theorem 9.18 (with $T=0$ there) holds, and Theorems 9.19 and 9.20 hold. Finally, if $T \leq T'$, then
 $C(T) \leq C(T')$ and $P(T) \leq P(T')$.

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Remark 9.22

Summary of the effect on the stock option price when the parameter is increased while the other ones are kept constant:

	EC	EP	AC	AP
K	-	+	-	+
S	+	-	+	-
T	?	?	+	+
r	+	-	+	-
D	-	+	-	+
σ	+	+	+	+

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