Math 5737/Econ 5337
Financial Mathematics

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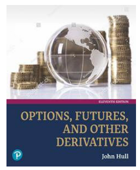

1

Literature

- Options, Futures, and other Derivatives, John C. Hull, Prentice Hall, 2022 (11th edition)
- Mathematics of Finance, Marek Capiński and Tomasz Zastawniak, Springer, 2011 (2nd edition)
- Risk-Neutral Valuation, Nick Bingham and Rüdiger Kiesel, Springer, 2004 (2nd edition)

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Chapter 1

Introduction

3

Definition 1.1

A derivative (derivative security, contingent claim) is a financial instrument whose value depends on (derives from, is contingent on) the values of other, more basic, underlying variables (the underlying).

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Example 1.2

- Forwards
- Futures
- Swaps
- Options

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Remark 1.3

Market forms for derivatives are

- Exchange-traded markets, e.g.,
  NYSE 1792, CBOT 1848, CME 1919, NASDAQ 1971, CBOE 1973
  - open outcry system
  - electronic
- Over-the-counter markets

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Definition 1.4

There are three types of traders:

- Hedgers attempt to reduce exposure to risk a company already faces
- Speculators invest available funds opportunistically in the hope of making a profit
- Arbitrageurs try to lock in riskless profit

  arbitrage: risk-free profit with no initial investment

  no-arbitrage principle: arbitrage opportunities are absent

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Definition 1.5

- A forward contract is an agreement to sell or buy an asset at a fixed date in the future (delivery time) for a price specified in advance (forward price, delivery price).
- The party selling the asset assumes what is termed a short (forward) position, while the party buying the asset enters into a long (forward) position.

Definition 1.5 (continued)

- It costs nothing to enter a forward contract. A forward contract can be contrasted to a spot contract, which is an agreement to buy or sell an asset today.

Example 1.6 (a)

Suppose that

- The spot price of 3 grams of gold is \$420
- The 1-year forward price of 3 grams of gold is \$450
- The 1-year interest rate is 5% p.a.

Example 1.6 (b)

Suppose that

- The spot price of 3 grams of gold is \$420
- The 1-year forward price of 3 grams of gold is \$430
- The 1-year interest rate is 5% p.a.

Example 1.7

Foreign exchange quote for GBP in USD
on August 15, 2026

	buy	sell
spot	1.35584	1.35617
1-month forward	1.35627	1.35661
3-month forward	1.35660	1.35698
6-month forward	1.35652	1.35695

Example 1.7 (continued)

(a) Say it is August 15, 2026, and Import Co, a company based in the US, knows that it will have to pay GBP 10 million on November 15, 2026, for goods purchased from a British supplier.

(b) Export Co exports goods to UK and knows it will receive GBP 10 million in three months.

Definition 1.8

- A European call option is a contract giving the holder the right (but no obligation) to buy an asset (the underlying) for a price fixed in advance (exercise price, strike price) at a specified future time (exercise time, expiry time, maturity).
- A European put option is a contract giving the right to sell an asset for a certain strike price at a certain exercise time.

Definition 1.8 (continued)

- Since payoffs are nonnegative, a premium (the market price of the option) must be paid when acquiring an option.
- An American put or call option can be exercised any time up to and including the expiry time.

Example 1.9

Suppose an investor owns 1000 Microsoft shares on August 15, 2026, \$500 per share is the current price. Suppose the investor is concerned about a possible share price decline in the next 2 months and wants protection. The investor could buy 1000 put options with strike price \$515 and expiration time October 17, 2026. Assume each option price is \$20. Ignore the time value of money.

Example 1.10

On August 15, 2026, European calls on Microsoft stock with strike price 515 USD and exercise time October 17, 2026 were traded at 18.72 USD at NASDAQ. Ignoring the time value of money, when will the investment bring profit?