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Chapter 9

Properties of Stock Options

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132

132

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Proposition 9.1

If we let c, p, C, P be the prices of a European call, European put, American call, American put, respectively (all on the same stock, same K and same T), then

$$0 \leq c \leq C \text{ and } 0 \leq p \leq P.$$

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133

133

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Theorem 9.2

For a stock paying no dividends,

$$(S_0 - Ke^{-rT})^+ \leq c < S_0.$$

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134

134

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Example 9.3

For a European call on a non-dividend paying stock with $S_0 = 51$, $K = 50$, $T = 1/2$, $r = 0.12$, find the bounds on c .

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135

135

Theorem 9.4

For a stock paying no dividends,

$$(Ke^{-rT} - S_0)^+ \leq p < Ke^{-rT}.$$



Example 9.5

For a European put on a non-dividend paying stock with $S_0 = 38$, $K = 40$, $T = 1/4$, $r = 0.1$, find the bounds on p .



Remark 9.6

- Theorems 9.2 and 9.4 can be graphically summarized
- Note also that by Corollary 5.12 (first part), $S_0 - Ke^{-rT} = V(0)$ is the value of a long forward contract on the non-dividend paying stock



Theorem 9.7

Put-call parity holds:

$$c - p = S_0 - Ke^{-rT}.$$



Example 9.8

Suppose a stock paying no dividends trades at \$15.60 a share. European calls on the stock with strike price \$15 and exercise date in 3 months are trading at \$2.83, $r = 6.72\%$ (cc). What is the price of a European put on the same stock with the same strike price and the same exercise date?



Theorem 9.9

For a stock paying no dividends,

$$C = c.$$



Corollary 9.10

For a stock paying no dividends,

$$(S_0 - Ke^{-rT})^+ \leq C < S_0.$$



Example 9.11

Suppose $S_0 = 10$, strike price of American put expiring in one year is \$80, and $r = 0.16$. Here it is better to do early exercise.



Theorem 9.12

For a stock paying no dividends,

$$(K - S_0)^+ \leq P < K.$$



Theorem 9.13

Put-call parity estimates hold:

$$S_0 - K \leq C - P \leq S_0 - Ke^{-rT}$$

(for a stock paying no dividends).



Example 9.14

Suppose $S_0 = 19$, $K = 20$, $T = \frac{5}{12}$,
 $r = 0.1$, $C = 1.50$. Find the
 bounds on P .



Theorem 9.15

For a stock paying dividends with
 present value D ,

- $(S_0 - D - Ke^{-rT})^+ \leq c < S_0 - D$
- $(Ke^{-rT} + D - S_0)^+ \leq p < Ke^{-rT}$
- $c - p = S_0 - D - Ke^{-rT}$



Theorem 9.15 (continued)

- $S_0 - D - K \leq C - P \leq S_0 - K e^{-rT}$
- $\max\{0, S_0 - D - K e^{-rT}, S_0 - K\} \leq C < S_0$
- $\max\{0, K e^{-rT} + D - S_0, K - S_0\} \leq P < K$



Remark 9.16

In the rest of this chapter, we discuss the dependence of c , p , C , P on K , S , T , and r (varying only one variable while keeping the others constant; all on stocks paying no dividends).



Proposition 9.17

c is nonincreasing in K while p is nondecreasing in K , i.e.,

$$c(K) \geq c(K') \text{ and } p(K) \leq p(K')$$

for

$$K \leq K'.$$



Theorem 9.18

c and p are Lipschitz continuous in K , namely,

$$|c(K) - c(K')| \leq e^{-rT} |K - K'|$$

and

$$|p(K) - p(K')| \leq e^{-rT} |K - K'|.$$



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Theorem 9.19

c and p are **convex** in K , i.e.,
 $K \leq K', \tilde{K} = \alpha K + (1 - \alpha)K', 0 < \alpha < 1$
implies
 $c(\tilde{K}) \leq \alpha c(K) + (1 - \alpha)c(K')$
and
 $p(\tilde{K}) \leq \alpha p(K) + (1 - \alpha)p(K')$.



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152

152

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Theorem 9.20

If $S \leq S'$, then

- $c(S) \leq c(S')$ and $p(S) \geq p(S')$
- $c(S') - c(S) \leq S' - S$
- $p(S) - p(S') \leq S' - S$
- c and p are **convex** in S .



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153

153

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Theorem 9.21

For American options,
Proposition 9.17 holds, Theorem
9.18 (with $T = 0$ there) holds,
and Theorems 9.19 and 9.20
hold. Finally, if $T \leq T'$, then
 $C(T) \leq C(T')$ and $P(T) \leq P(T')$.



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154

154

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Remark 9.22

Summary of the effect on the stock option price when the parameter is increased while the other ones are kept constant:

	EC	EP	AC	AP
K	-	+	-	+
S	+	-	+	-
T	?	?	+	+
r	+	-	+	-
D	-	+	-	+
σ	+	+	+	+



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155

155