29. Suppose X is adapted and let B be a Borel set. Show that

$$\tau = \inf\{n \in \mathbb{N}_0 : \ X_n \in B\} \quad \text{is a stopping time.}$$

30. Suppose τ and σ are stopping times. Show that
    (a) $\sigma \wedge \tau$ and $\sigma \vee \tau$ are stopping times,
    (b) $\mathcal{F}_{\sigma\wedge\tau} = \mathcal{F}_\sigma \cap \mathcal{F}_\tau$.
31. Prove Lemma 4.7 from the Lecture Notes.
32. Prove Doob's STP if X is a supermartingale.
33. Prove Doob's OST if X is a supermartingale.
34. Consider an American put with expiration time 2 and strike price 5 in the BAPM with $N = 2$, $\tilde{p} = \tilde{q} = 1/2$, $r = 1/4$, $u = 2$, $d = 1/2$, $S_0 = 4$. Let Y_k be the maximum of zero and the payoff if the put is exercised at k. Let X be the discounted Y process.
    (a) Is τ defined by $\tau(UU) = \tau(UD) = 2$, $\tau(DU) = \tau(DD) = 1$ a stopping time? If so, find $\mathcal{F}_\tau$, X_τ, X^τ, and $\tilde{\mathbb{E}}(X_\tau)$.
    (b) Is ρ defined by $\rho(DD) = 2$, $\rho(UU) = \rho(UD) = \rho(DU) = 1$ a stopping time? If so, find $\mathcal{F}_\rho$, X_ρ, X^ρ, and $\tilde{\mathbb{E}}(X_\rho)$.