

48. Let  $X(t) = -1$  for  $0 \leq t < 1$ ,  $X(t) = 1$  for  $1 \leq t < 2$ , and  $X(t) = 2$  for  $2 \leq t \leq 3$ .

Find

$$\int_0^3 X(t) dW(t)$$

and its distribution.

49. Finish the proof of Theorem 6.4 if  $l = k$ .

50. Let  $W$  be Brownian motion. Verify directly that

$$\int_0^t s dW(s) = tW(t) - \int_0^t W(s) ds.$$

51. Let  $W$  be Brownian motion. Verify directly that

$$\int_0^t W^2(s) dW(s) = \frac{1}{3} W^3(t) - \int_0^t W(s) ds.$$

52. Let  $W$  be Brownian motion. Show that

$$\mathbb{E} \left( \left( \int_0^T W(s) dW(s) \right)^2 \right) = \int_0^T \mathbb{E}(W^2(s)) ds.$$

53. Use Itô's formula to find an expression for  $\int_0^T W^m(t) dW(t)$  for  $m \in \mathbb{N}$ .

54. What does Itô's formula for BM say if  $f(t, x) = e^{\lambda x - \frac{\lambda^2 t}{2}}$  with  $\lambda \in \mathbb{R}$ ?

55. Let  $W$  be Brownian motion. Use Itô's formula to find

(a)  $de^{W(t)}$ ;

(b)  $d \sin(W(t))$ ;

(c)  $d \cos(W(t))$ ;

(d)  $de^{iW(t)}$ .