Predicting Response of Risk-Seeking Systems during Project Negotiations in a System of Systems

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***Abstract*—During project negotiations, typically, the awarding agency seeks bids from multiple parties. Examples of this setting include firms no longer willing to produce parts in-house or an airport seeking contracts for renovation. Risk-*seeking* parties are those that agree to work with *lower* budgets and under *shorter* deadlines, while risk-*averse* parties exhibit the opposite behavior. This setting can be found in the context of System of Systems (SoS), where the SoS coordinator (the firm) has access to behavior characteristics of individual systems (parties) and their current workload from past interactions. The problem we study is for the SoS coordinator to predict the response of the systems in terms of budgets, deadlines, and performance targets – in advance of obtaining the actual response. This prediction can help the SoS negotiate the best deal. We present a quantitative model that predicts this response. Our model employs Markov chains to capture dynamics of the project, which would result when a bid is won, to quantify the response. Further, our model accounts for the risk-taking tendencies and agility of the firm. We also analyze mathematical properties and provide numerical results to illustrate how our model can be used in a negotiation process.**

***Index Terms*—entrepreneurial risk; System of Systems (SoS); project negotiations; Markov chains; budgets**

I. INTRODUCTION

In this paper, we present a fully observable Markov chain model to study the dynamics of the bidding process and the negotiations and re-negotiations that ensue once the bidding process starts in projects related to outsourcing work. The specific example (case study) that we focus on is one commonly studied in System of Systems (SoS) theory, which has been widely applied in the defense industry and where outsourcing projects are very common. However, the model we present here is of a general nature and will be applicable to any firm seeking to outsource a part of its work to external agencies. Further, in this paper, we will study **risk-seeking** behavior from the parties. By risk-seeking, we mean the party has traits linked to embracing and attracting risk. A risk-seeking agent is hence often willing to perform a task faster and under lower budgets than a risk-averse one. Naturally, a risk-seeking agent accepts the dangers that come with having to meet shorter deadlines and working with fewer resources. Clearly, there are chances of failure for which the risk-seeking agent may face penalties. **Risk-averse** agents, on the other hand, are those that avoid dangers of the type mentioned above, and prefer longer deadlines and greater access to resources. Naturally, risk-averse agents also usually have a lower probability of winning the bid.

It is commonplace in industry to invite bids from external agencies/parties for tasks that a firm no longer wants to carry out itself. Often, this is driven by economic needs and the fact that the external parties chosen, being specialists in that task, can perform the same task more efficiently and at a lower cost. The task can be, for instance, the production of a part for which the manufacturing technology has recently become expensive, or the delivery of a service, e.g., delivering packages to a location not affordable any more to a mail carrier (e.g., UPS or FedEx). These tasks can be viewed as "projects" in a firm. Typically, such projects start with the firm asking for bids from other parties (contractors), which leads to negotiations and re-negotiations (see e.g., Fudenberg and Tirole [6]), and end with the firm awarding the contract to one of the contractors.

Now consider a scenario in which past experience of the firm (with performing the task) suggests that the task should be completed in four weeks. Two contractors (systems), X and Y, approach the firm with proposals. X is willing to complete the project in five weeks while Y is willing to do the same in three. Costs are roughly proportional to the duration of the project. If everything else about the proposals is the same, typically, the preference will be for Y because it proposes to complete the project in a shorter duration of time and also charges less money. However, what can really be helpful to the firm is knowledge — *in advance* of the negotiations — about estimates of project-completion times and budgets that each system would propose. This is not only because the firm would like to make a profitable deal in the bargaining process, but also because variability in the project completion times can cause delays to the firm in meeting its own deadlines.

The scenario we study is typical of what often happens in industry, under the following assumption: The firm has past experience of working with both systems, X and Y, and has data or other tangible pieces of information that allow it to estimate (i) the willingness of each system to undertake Entrepreneurial Risk (ER) and (ii) the workload to which the system is subjected. To be more specific, by willingness to undertake ER, we mean how eager the party is to please the firm and get future work from it. This willingness is often determined by how "agile" the system is and the budget it is willing to work under. The agility of a system can be measured via probabilistic estimates of weekly (or fortnightly or monthly) completion of each sub-task (phase) within the project; such estimates are usually available to the firm from past interactions. The budget with which a system will be willing to work either equals the budget offered by the SoS

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or is a fraction of it. Again, based on past experience, it is also possible for the firm to predict the value of this fraction. The ER factor is usually a key determinant of the outcome of a negotiation and will be hence an important consideration in this paper. Workload, on the other hand, typically depends on how busy the system is currently with other work, and about this, sometimes, only partial information is available to the firm. An exception to this is a scenario in which the systems regularly serve as vendors to the firm and are willing to share all their information with the firm.

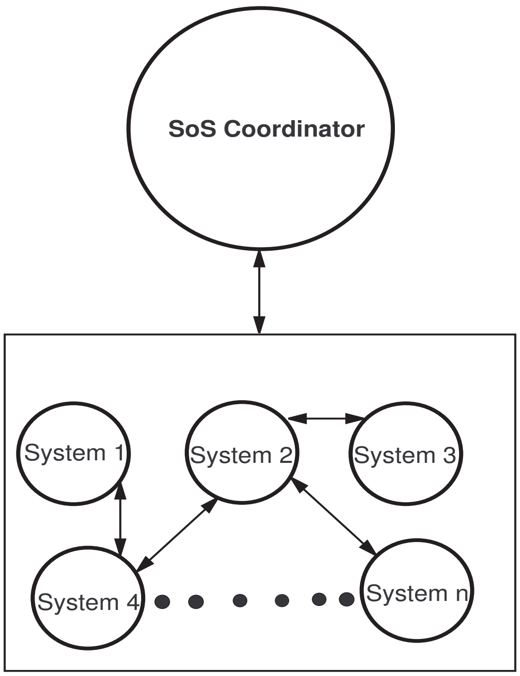

Fig. 1. SoS and the interacting systems via a coordinator

Under the conditions described above, the firm is often interested in gaging how the individual systems (contractors) will respond in the bidding process. This framework of decision-making can be studied under the general umbrella of System of Systems (SoS), where each individual contractor is viewed as a system, and the firm is the coordinator of the SoS [1]. See Figure 1 for a pictorial representation of this framework. The SoS coordinator works together with the other systems, who may or may not be working with each other; typically negotiations and renegotiations occur between the SoS coordinator and the individual systems. In the negotiation process, the SoS coordinator often attempts to second guess what bids the systems will provide. This is because there is incomplete information, and the firm wishes to use bargaining that occurs within the negotiation and re-negotiation processes to its own advantage. To be more specific with respect to the problem we study, the SoS controller provides inputs to each system in regards to budgets, performance targets (deliverables), and deadlines. Each system considers the offer and returns to the coordinator with its own budget requirements, the performance targets that it can meet, and the deadline by which it can deliver. We provide a model that will allow the firm (SoS coordinator) to estimate what bids it can get from the systems — depending on the information it has gathered about their risk and workload. The bids will thus have three descriptive qualifiers: (a) performance (deliverables), (b) budget, and (c) deadline.

Just as the firm has partial information about the systems, each system in turn does not have perfect information about the level of funding (the budget) the firm can provide. Hence, the system guesses estimates of this budget in its own calculations. In the real world, negotiations frequently occur under such conditions, where both parties are playing a guessing game (obviously an intelligent one rather than one based on wild guesses) rooted in estimates they have about each other. As such, in the negotiation process, it is helpful for a firm to predict what budget, for example, the system responds with; the system's desired budget in turn depends on what the system's guess of the budget of the SoS to be. The main ideas underlying the negotiation modeling that we study in this paper are shown via Figure 2.

Entrepreneurial Risk and its Role in Business: As stated above, a risk-seeking system is often willing to perform the same tasks *faster* and at budgets *lower* than those required by the competition. It can achieve this via a variety of internal mechanisms, such as a novel business plan that results in lower manufacturing costs, lower holding costs, and/or lower shipping costs. A well-known example of this includes Dell's entry into the personal computer business in the early part of the century via a bold initiative of shipping computers by air to the customers' doorstep [8]. It was able to achieve that — even at a discount price initially — because it did not own expensive warehouses or retail stores, and customers had to order the product online; in other words, its business model dramatically shrank inventory storage costs and was thereby able to pass on the reduced costs to customers in the form of discount prices and rapid delivery. Another example is that of Southwest Airlines which gradually captured a significant portion of the domestic market in the U.S. by offering discount fares. Southwest Airlines did not run on the hub-and-spoke model, saved significant amounts of money by flying through smaller airports, and passed on the profit margins from the reduced costs to customers as discounts. Even to this day, Southwest, unlike its competition (network airlines), can afford to allow each customer two free bags because of its novel business model which is said to have produced the so-called "Southwest effect"[16]. In the business world, it is the ability to take on risky initiatives that produce lower costs and lead times that determines the success of many firms. It is in the process of negotiating contracts that the role of these risky initiatives comes into play and determines the final outcomes.

Literature Review: The notion of ER and the impact on budgets which the entrepreneur is willing to work under has been studied in economic theory for many years. Steinhaus [14] originally considered a related problem in the context of division of labor. Recently, ER has also been studied in the literature (see [9] and the references therein) as a productivity (agility) indicator and a willingness to work under lower budgets. Recently, authors of books have started publishing on their own, bypassing publishers, and are selling books online to a variety of sellers (systems), e.g., amazon.com, Barnes and

Noble, etc. This is an interesting phenomenon where the ER of amazon.com (system in this context) is helping the SoS (author in this context) succeed [20]. Most airports face renovation projects, and governments spend money on transportation infrastructure projects from time to time; many of these projects result in contracts offered to private companies, which can be studied via the SoS paradigm. The SoS paradigm is also studied in the defence setting [5] and multi-agent systems that have been studied widely in the recent past (see [2], [17], and [3]). An example of a multi-agent setting can be found in a wireless service network that uses different "cells" at different points of time to minimize the probability of dropped calls, where each cell can be viewed as a system (automaton), and the controller is aware of how much each cell is loaded and its capabilities [7].

There is a significant body of literature on the topic of managing risk in the enterprise context; see Choi et al. [4] for a detailed survey. Risk management is naturally of strong interest in banking ([19]) and vendor selection in supply chains ([13]). But our paper differs from the viewpoint adopted in much of the literature cited above, where risk is viewed as undesirable and something to be minimized. On the contrary, in our setting, the ability to take on risk is actually a desirable characteristic within the enterprise—a trait that can lead entrepreneurs to success and one that is especially relevant to systems engineering when numerous systems interact with each other. In the context of related work on negotiations and projects, we need to discuss three papers. Murtoaro and Kujala [10] state that although the firm and the systems (contractors) face numerous difficulties in the negotiation of projects, the topic of quantitative models for project negotiations has not attracted much research interest in the literature. They also point out that what one witnesses in the real world is a "recurring continuum of negotiations;" however, no model is presented by them to capture in a numeric form the reaction of a system in the negotiation process. Miller et al. [9] discuss enterprise risk from the perspective of budgets and productivity, but the scope of their work does not extend to the problem studied here. And finally, the pioneering work of Fudenberg and Tirole [6] does lay the foundation for the theory of negotiations and re-negotiations, but does not apply it to the domain that we cover here.

Contributions of this paper: This paper seeks to present, to the best of our knowledge for the first time, a model for the negotiation process — based on partial information about an individual system and the system's guessed estimates about the SoS. In particular, the goal is to predict how the system will react to a given set of three variables, namely, a budget, performance requirements, and deadlines, from the SoS, when the SoS invites negotiations for a project. The reaction of the system will also be in terms of these three variables that it returns with to the SoS. The associated model that we develop will use a guessed estimate of system characteristics, namely the ER parameter, and the value of the workload, as well as the total budget of the SoS, as estimated by the system. Further, our model will be based on a fully observable Markov chain, which can be constructed on the basis of the workload and the weekly agility parameters of the system.

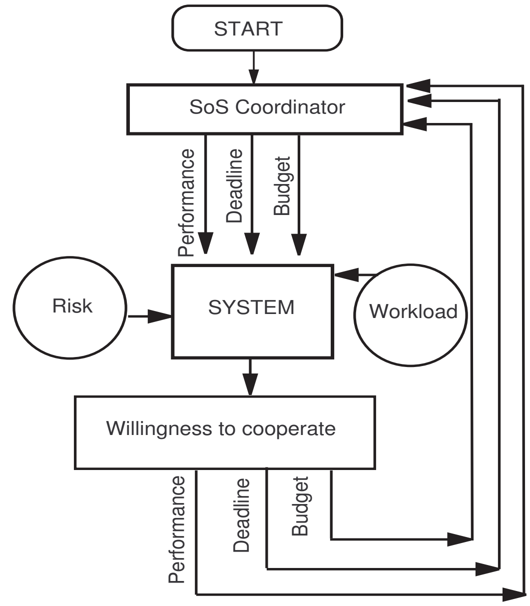

Fig. 2. The negotiation modeling process involving the SoS and the individual systems

The model serves as an alternative to simulation modeling, which generally consumes more time to run and may require additional input data. Finally, our model seeks to link ER to behavior in project negotiations, which we also believe is a new contribution that will stimulate further research in modeling risk of SoS behavior.

The rest of this paper is organized as follows. Section II presents the underlying Markov chain model. Section III analyzes key mathematical properties of the model that provide useful insights for the negotiation modeling, while Section IV performs numerical experiments with the Markov chain model to illustrate its usefulness. Section V ends this paper with conclusions and topics for future research.

II. A Markov Chain Model

The workload assigned by the SoS to each system will be assumed to be a project that takes a random amount of time to complete. The internal phases of this project will be modeled as the states of a Markov chain. Since information about the phase of the project is known with certainty, we have a fully observable Markov chain. The notion of modeling transitions from one phase to another until the system is absorbed into an absorbing state can be found in marketing management literature [11]. The goal is to exploit the mathematics of the Markov chain to estimate the expected (mean/average) duration of the project and the funds needed by the system. Via an absorbing Markov chain model, the expected duration is estimated. An upper limit on the duration is also computed assuming that the system is loaded to its full capacity. The willingness to cooperate, which seeks to encapsulate the idea of how eager the system at hand will be willing to take on the new task, given its own workload, is then estimated. This willingness to cooperate allows us to compute the deadline to which the system will agree as well as the budget expectations that the system will have. A system prone to taking ER will accept a lower budget in order to obtain the contract while one that is averse to ER will require higher budgets.

We begin by formally presenting key assumptions about our model.

1) The internal phases can be modeled as states of a Markov chain such that the transitions from one phase to another satisfy the Markovian property.
2) The SoS under consideration is a closed one, i.e., each system interacts only with other systems with the SoS.
3) The transition probability matrix of the underlying Markov chain is static.

We provide in the Appendix an alternative model that can work when the first and the third assumptions above are relaxed, i.e., the transitions are not Markovian. The second assumption is somewhat restrictive in the sense that if we have systems interacting with systems outside the SoS, those interactions can affect the workload. However, relaxing this assumption is beyond the scope of this work.

We now present with some notation. The following will be the input parameters in our model.

- n: number of systems in the SoS
- i: index of the system under consideration, where $i = 1, 2, \ldots, n$
- k_i: the number of systems the ith system interacts with
- p_i: performance target desired from the ith by the SoS
- D_i: delivery deadline set for the ith system by the SoS
- f_i: funds allocated to the ith system by the SoS
- η: ER budget parameter $\in$ $(0, 1]$, where 0 indicates extremely willing to take ER (i.e., risk-seeking) and 1 being very risk-averse
- l and m: agility parameters, each taking values in the interval (1,2] with 1 being very sluggish and 2 being very agile
- β: parameter used by system to estimate the total budget of the SoS, where $\beta > 0$

We note that agility parameters, l and m, are often measured in the interval $(0, 1]$ (see [15]). Our model requires that the value be projected by linear interpolation onto the interval $(1, 2]$, since both of these agility parameters are used in the denominator of a term that represents a probability (see Equation (1) below), which cannot exceed 1. The term denoting probability in our model can exceed 1 if the agility parameter's value is less than 1 (and greater than 0). Since this is not an acceptable situation, it is necessary for the agility parameter to exceed 1, and therefore, we set the lower limit to 1, via the interval $(1, 2]$. This leads to an upper limit of 2; in general, however, any interval $(a, b]$, where a and b are strictly positive, that satisfies the property $(b - a) = 1$ will work for our model.

Obtaining values of the inputs in a real-world project: For using our model, it will be necessary to obtain values for the input parameters discussed above. All of the inputs above, except for a few, depend on the project, and can be obtained from the system under consideration. For instance, the delivery deadline, funds allocated etc are parameters for which the project manager has accurate data. However, the following four parameters will need to be estimated from past experience: η, β, and the agility parameters, l and m. The agility parameters can be estimated from past interaction with the system (see e.g., [15]). The ER budget parameter, η, will depend on how the SoS views the system in question. Again, this will depend on past interaction. A system that is known to be unwilling to take on risk should be assigned a value close to 1, while one that is known to be risk-seeking should be assigned a value close to 0. Systems in between the two extremes should be assigned values somewhere in the middle of the range. In business deals, it is common to make an assessment of the risk-seeking attitude of the partner; our ER budget parameter requires a similar prior analysis of the partner. This has also been discussed in the literature on supply chain management in aspects other than risks, e.g., reliability (of vendors) [13]. Further, for measuring entrepreneurial risk, η, there is a growing body of literature (see [9] and references therein), and like for the agility parameters, any scale, e.g., the Likert scale, can be used to measure it; what is necessary for use in our model is that the value of η be mapped, via normalization, into an interval $(0, 1]$. Since both agility and ER will be estimated by humans in the negotiation process, surveys can be used to measure these parameters, and hence, it is quite possible that a Likert scale, which is popular in surveys, may be used to capture the raw data on these parameters. Lastly, the value of β is dependent on the system's perception of how much budget is available with the SoS. In many real-world projects, most bidders have a good idea of how much the actual budget will be, and hence can compute its value based on past experience. For instance, when a bidder estimates the total budget for an airport renovation project, he/she has a rough idea of the total amount the airport has allocated for the project. In the past if the airport has spent \$50,000 for a similar project, but the current advertisement from the airport says that \$40,000 have been assigned as the maximum amount that can be spent, then $\beta = 50,000/40,000 = 1.25$. When no past data is available, β should be set to 1. Finally, we must note that values of some of these parameters may not be accurate, but in negotiations, one must work with imperfect information, as discussed in the Nobel-prize-winning work of [6].

The outputs that our model will generate include the following:

- ϕ: the willingness to cooperate
- τ_i: the expected duration of the project (task) that the ith system will require
- $\tau_{\max}$: estimate of an upper bound on the duration required by any system for the project
- D'_i: the deadline agreed to by the ith system in the negotiation
- p'_i: the performance target agreed to by the ith system in the negotiation
- f'_i: the budget offered by the ith system in response to the negotiation process

The three internal phases in the project will be assumed to be: kickoff, mid-term review, (which can be a design review for defense projects) and completion. In the Appendix, we show how this model can be easily extended to projects with additional phases. Each of these will be considered to be states in an absorbing Markov chain in which the absorbing state is completion. We will assume that after unit time (e.g., one cycle), the project will move from one state to the next with a fixed probability that will depend on the agility of the system as well as how loaded it is. This is typical of most projects where each sub-task is of a duration of a cycle, which can be a week, a fortnight, or a month. Past experience with the project can help one estimate the probability that each phase will be a completed in a cycle. We will refer to these probabilities as the phase-completion probabilities, and these will be estimated using the agility parameters. The Markov chain of the internal system dynamics will be defined by the following transition probability matrix:

$$\mathbf{P} = \begin{bmatrix} \frac{k_i}{ln} & 1-\frac{k_i}{ln} & 0 \\ 0 & \frac{k_i}{mn} & 1-\frac{k_i}{mn} \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

where state 1 denotes the kickoff meeting, state 2 denotes the mid-term review, and state 3 denotes the completion. Here, the phase completion probability of the first phase (from the kickoff to the mid-term review) will be given by $(1-\frac{k_i}{ln})$ and the same for the second phase (from the mid-term review to the completion) will be given by $(1-\frac{k_i}{mn})$. Note that l and m are the agility parameters for the first and second phase respectively. The higher the values of the agility parameters, the sooner the phase will be completed. Also, the lower the value of k_i, the sooner the phase will be completed. We will explain this mathematically and via a numerical example later. The Markov chain is depicted pictorially via Figure 3.

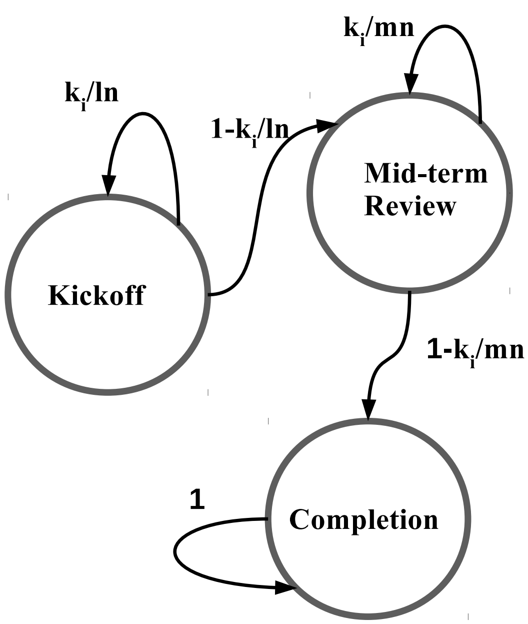

Fig. 3. The Markov chain model for the project

We now present the following result that allows us to compute the expected time to complete the entire project:

Theorem 1. *The expected time for completion of the project by the ith system, τ_i, can be expressed in terms of l, m, k_i, and n as follows:*

$$\tau_i = \frac{ln}{ln-k_i} + \frac{mn}{mn-k_i}. \tag{2}$$

Proof: Using the analysis of an absorbing Markov chain, the expected number of transitions to reach the absorbing state starting from state 1 can be computed as follows [12]. We begin by eliminating the absorbing state from the transition probability matrix, $\mathbf{P}$, and the result is:

$$\mathbf{Q} = \begin{bmatrix} \frac{k_i}{ln} & 1-\frac{k_i}{ln} \\ 0 & \frac{k_i}{mn} \end{bmatrix}.$$

Then, if we perform the matrix operation

$$\mathbf{M} = \mathbf{I} - \mathbf{Q} = \begin{bmatrix} 1-\frac{k_i}{ln} & \frac{k_i}{ln}-1 \\ 0 & 1-\frac{k_i}{mn} \end{bmatrix},$$

where $\mathbf{I}$ is the identity matrix and define

$$\vec{c} = \mathbf{M}^{-1}\vec{w}$$

where $\vec{w}$ is a column vector of ones, then $\vec{c}$ is a column vector of the expected transition times from states 1 and 2 [12]. Thus, the expected transition time to absorption from state 1 (initial phase) is $c(1)$, where $c(i)$ denotes the ith element of the column vector $\vec{c}$. Using matrix inversion, after some elementary algebra, we can show that:

$$\mathbf{M}^{-1} = \frac{1}{(1-\frac{k_i}{ln})(1-\frac{k_i}{mn})} \begin{bmatrix} 1-\frac{k_i}{mn} & 1-\frac{k_i}{ln} \\ 0 & 1-\frac{k_i}{ln} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{1-\frac{k_i}{ln}} & \frac{1}{1-\frac{k_i}{mn}} \\ 0 & \frac{1}{1-\frac{k_i}{mn}} \end{bmatrix}.$$

Then,

$$\vec{c} = \begin{bmatrix} \frac{1}{1-\frac{k_i}{ln}} & \frac{1}{1-\frac{k_i}{mn}} \\ 0 & \frac{1}{1-\frac{k_i}{mn}} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{1-\frac{k_i}{ln}} + \frac{1}{1-\frac{k_i}{mn}} \\ \frac{1}{1-\frac{k_i}{mn}} \end{bmatrix},$$

which implies that $\tau_i = c(1) = \frac{1}{1-\frac{k_i}{ln}} + \frac{1}{1-\frac{k_i}{mn}} = \frac{ln}{ln-k_i} + \frac{mn}{mn-k_i}$. ■

a) Computing the willingness to cooperate: Setting $k_i = n$ and searching over all feasible values for l and m, one can obtain an upper bound on the expected number of cycles needed for completion. This is because setting $k_i = n$ indicates that the system has the highest possible workload, i.e., it is interacting with all the other systems. This upper bound captures the longest amount of time the project can take to complete. But the upper bound may even be higher than the one obtained in this process, and an estimate may be available from past experience. We will assume that an estimate of this upper bound, denoted by $\tau_{\max}$, is available to us. For the ith system, the willingness to cooperate can then be computed as the following probability:

$$\phi = 1 - \frac{\tau_i}{\tau_{\max}}. \tag{3}$$

Note that this probability will equal 1 when the project takes the least amount of time to complete (theoretically zero) and will equal 0 when the project takes the longest amount of time needed for completion ($\tau_{\max}$). The willingness to cooperate is indirectly a measure of the risk-seeking ability of the system. As shown below, out model hinges on computing this crucial parameter. Intuition suggests that the willingness to cooperate should decline as the project duration increases. Further, the willingness to cooperate should depend on how agile the system is and how loaded it is with respect to interaction with the other systems. We shall make these notions mathematically precise later.

b) Deadline accepted by system and performance: Typically, the willingness to cooperate and the actual project completion time together determine the deadline by which the system will be able to produce the project deliverables. Hence, we propose the following relationship for the deadline that the system will return with:

$$D_i' = \max\left\{\frac{D_i}{\phi}, \tau_i\right\}. \tag{4}$$

Further, the actual performance of the system will be given as

$$p_i' = \phi \; p_i \tag{5}$$

which is a fraction of the maximum performance that can be expected from the system.

c) Budget and entrepreneurial risk: The system has to guess what the total budget of the SoS is, and it will use the following formulation for the upper limit on the budget:

$$\bar{B} = f_i \beta \tag{6}$$

where β is an estimation parameter for the total budget of the system. On the basis of this, a conservative estimate of the time rate at which system can be funded will be (in units of dollars per unit time):

$$\rho = \frac{\bar{B}}{\tau_{\max}}. \tag{7}$$

Then, the maximum funding that the system can draw will be given as: $\rho\tau_i$. A system willing to take ER will then charge a fraction of this amount:

$$f_i' = \eta\rho\tau_i, \tag{8}$$

where η is an ER budget parameter, which equals the fraction discussed above. Note that in the model above, we did *not* use the actual budget of the SoS that it obviously has access to; rather it uses the estimate generated by the system. This is because the goal of the model is to predict how the system will respond, and since the system does not have direct knowledge of the budget, the model must use the estimate that the system has of the budget of the SoS.

III. Analytical Insights

We now prove three key properties of the output parameters of our model. In particular, we study analytically the impact of the values of the agility parameters and the workload on the willingness to cooperate as well as the budget with which the system returns to the SoS. Intuition suggests that as the agility increases, the willingness to cooperate should increase and the budget should decrease. Our aim here is to make these properties mathematically precise. Our first result is in regards to the agility parameters. Note from Equation (3) that

$$\frac{\partial \phi}{\partial \tau_i} = -\frac{1}{\tau_{\max}}, \tag{9}$$

where $\tau_{\max} > 0$. Hence, we have that $\frac{\partial \phi}{\partial \tau_i} < 0$. We can now prove the following property of the agility parameters:

Theorem 2. *The willingness to cooperate is an increasing function of the agility parameters, l and m.*

Proof: We first consider the parameter l. From Equation (2), we have that:

$$\frac{\partial \tau_i}{\partial l} = -\frac{k_i n}{(ln - k_i)^2}, \tag{10}$$

which combined with Equation (9) leads to:

$$\frac{\partial \phi}{\partial l} = \frac{k_i n}{(ln - k_i)^2 \tau_{\max}}.$$

Since all terms in the partial derivative are positive, we have the result for l. Similarly, we can prove the result for m by showing the following:

$$\frac{\partial \phi}{\partial m} = \frac{k_i n}{(mn - k_i)^2 \tau_{\max}} > 0.$$

■

It is clear that a firm interested in negotiating a profitable deal needs to carefully examine the agility of the firm, since it is a prime indicator of its willingness to take risk as well as deliver on time. Our second result is for the workload parameter, k_i:

Theorem 3. *The willingness to cooperate is a decreasing function of the workload parameter, k_i.*

Proof: From Equation (2), we have that:

$$\frac{\partial \tau_i}{\partial k_i} = \frac{ln}{(ln - k_i)^2} + \frac{mn}{(mn - k_i)^2}, \tag{11}$$

which combined with Equation (9) leads to:

$$\frac{\partial \phi}{\partial k_i} = -\left(\frac{ln}{(ln - k_i)^2} + \frac{mn}{(mn - k_i)^2}\right)\frac{1}{\tau_{\max}} < 0.$$

■

The result above confirms our intuition that a system already burdened with work is less likely to seek work. We now show that the budget will decrease as the agility increases.

Theorem 4. *The budget, f_i', is a decreasing function of the agility parameters, l and m.*

Proof: We first consider the result for l. From Eqn. (8), we have that

$$\frac{\partial f_i'}{\partial \tau_i} = \eta\rho;$$

then applying the chain rule and from (10), we have that:

$$\frac{\partial f_i'}{\partial l} = \frac{\partial f_i'}{\partial \tau_i}\frac{\partial \tau_i}{\partial l} = -\eta\rho\frac{k_i n}{(ln - k_i)^2}.$$

Since all the parameters in the right hand side of the above are positive, we must have that f_i' is a decreasing function of l. Similarly, we can show that:

$$\frac{\partial f_i'}{\partial m} = -\eta\rho\frac{k_i n}{(mn-k_i)^2} < 0,$$

and we are done. ■

Again, the importance of the above result stems from the insight it provides in that a firm is likely to get work at a lower budget from a contractor if the contractor is agile, i.e., risk-seeking. These insights further strengthen our intuition that agility can be viewed as a dimension of risk that the system is willing to undertake.

IV. Numerical Results

We now illustrate our Markov chain model via numerical experiments. These experiments are designed to demonstrate that the values of performance targets, deadlines, and budgets can be computed easily in response to the same provided by the SoS. Further, our experiments also show that these computations can be easily incorporated into a negotiation modelling process.

In the first set of experiments we performed, the values of l and m were varied from 1 to 2, while the other inputs were set to values coming from a wide range — in order to test the robustness of our model. The inputs of our experiments are presented in Table I, where 20 different cases are generated. The value of $\tau_{\max}$ was set to 42 in our experiments. For each case, the outputs were computed as follows: τ_i from Equation (2), ϕ from Equation (3), f_i' from Equation (8), D_i' from Equation (4), and p_i' from Equation (5). The outputs for the 20 cases defined in Table I are provided in Table II. It must be noted that although we do not provide units for our outputs in the tables, because our model is general, in practice, deadlines and project completion times often have units of weeks, budgets are measured in dollars, while performance targets depend on the nature of the project. For a road construction project, for instance, the units of project deliverables can be in terms of miles of roads to be constructed.

We provide details of one specific randomly chosen numerical example, namely Case 17, as a vehicle to illustrate our model. There is nothing special about this case, and the results can be worked out for every other case in an analogous manner. As mentioned above, the inputs for the model are given in Table I and $\tau_{\max} = 42$. Using Equation (2), we have that

$$\tau_i = \frac{(1.8)(7)}{(1.8)(7)-5} + \frac{(1.2)(7)}{(1.2)(7)-5} = 4.13,$$

which can be verified as the entry for τ_i for Case 17 in Table II. We will similarly verify all other outputs shown in Table II for this case. Since $f_i = 32$ and $\beta = 4$ (both values are from Table I), via Equation (6), we have that

$$\bar{B} = (32)(4) = 128;$$

since $\tau_{\max} = 42$, we then have from Equation (7) that

$$\rho = \frac{128}{42} = 3.05.$$

To compute f_i', we use Equation (8):

$$f_i' = \eta\rho\tau_i = (0.6)(3.05)(4.13) = 7.55.$$

Also,

$$D_i' = \max\left\{\frac{16}{0.9}, 4.13\right\} = 17.74.$$

And finally,

$$p_i' = (0.9)(29) = 26.1.$$

TABLE I
Inputs for our numerical experiments

Case	l	m	n	k_i	η	β	p_i	f_i	D_i
1	1.1	1.1	8	6	0.3	2	30	21	10
2	1.2	1.2	12	10	0.5	5	25	45	20
3	1.3	1.3	16	9	0.7	7	16	23	12
4	1.4	1.4	14	9	0.9	3	14	34	15
5	1.5	1.5	13	10	0.2	4	28	21	12
6	1.6	1.6	21	7	0.8	5	32	60	14
7	1.7	1.7	22	5	0.3	6	33	34	12
8	1.8	1.8	9	7	0.8	3	27	23	16
9	1.9	1.9	10	8	0.2	2	28	25	12
10	1.1	1.9	11	10	0.4	3	31	35	6
11	1.2	1.8	17	4	0.6	7	36	32	15
12	1.3	1.7	15	3	0.7	6	32	29	10
13	1.4	1.6	12	5	0.5	2	33	20	11
14	1.5	1.5	15	4	0.2	5	23	40	12
15	1.6	1.4	17	7	0.9	4	27	53	14
16	1.7	1.3	11	8	0.4	7	26	54	15
17	1.8	1.2	7	5	0.6	4	29	32	16
18	1.9	1.1	9	3	0.3	5	25	30	17
19	1.1	1.1	15	7	0.5	3	29	29	12
20	1.1	1.1	20	9	0.8	6	34	25	11

TABLE II
Results of our numerical experiments

Case	τ_i	ϕ	ρ	f_i'	p_i'	D_i'
1	6.29	0.85	1.00	1.89	25.51	11.76
2	6.55	0.84	5.36	17.53	21.10	23.69
3	3.53	0.92	3.83	9.46	14.66	13.10
4	3.70	0.91	2.43	8.08	12.76	16.45
5	4.11	0.90	2.00	1.64	25.26	13.30
6	2.53	0.94	7.14	14.44	30.08	14.90
7	2.31	0.95	4.86	3.36	31.19	12.70
8	3.52	0.92	1.64	4.63	24.73	17.46
9	3.45	0.92	1.19	0.82	25.70	13.08
10	7.68	0.82	2.50	7.68	25.33	7.68
11	2.39	0.94	5.33	7.66	33.95	15.91
12	2.32	0.94	4.14	6.71	30.24	10.58
13	2.78	0.93	0.95	1.32	30.81	11.78
14	2.43	0.94	4.76	2.31	21.67	12.74
15	2.76	0.93	5.05	12.55	25.22	14.99
16	4.02	0.90	9.00	14.46	23.51	16.59
17	4.13	0.90	3.05	7.55	26.10	17.74
18	2.65	0.94	3.57	2.84	23.42	18.14
19	3.47	0.92	2.07	3.60	26.60	13.08
20	3.38	0.92	3.57	9.67	31.26	11.96

We also study the impact of changing the agility parameters, l and m, on the model outputs. The results are plotted, with the agility parameter on the x-axis, which is the independent variable, while the dependent variables are plotted on the y-axis. Figures 4 and 5 demonstrate the impact of l and m on ϕ the willingness to cooperate; in each figure, the independent variable was varied keeping all the other inputs constant. From

Figures 4 and 5, it is clear that as the values of the agility parameters increase, i.e., agility itself is improved, so does the willingness to cooperate. These figures also serve as a numerical verification of Theorem 2. Figures 6 and 7 seek to track the influence of the agility on the budget proposed by the system. These figures show that a more agile (risk-seeking) system, i.e., with a *higher* value for the agility parameter, would be willing to perform at a *lower* budget. These figures also numerically verify Theorem 3. Similarly, Figures 8 and 9 show that *higher* the agility parameter, the *shorter* the deadline to which a system will agree. Finally, Figures 10 and 11 demonstrate that the performance targets also improve as the agility parameters l and m, respectively, increase.

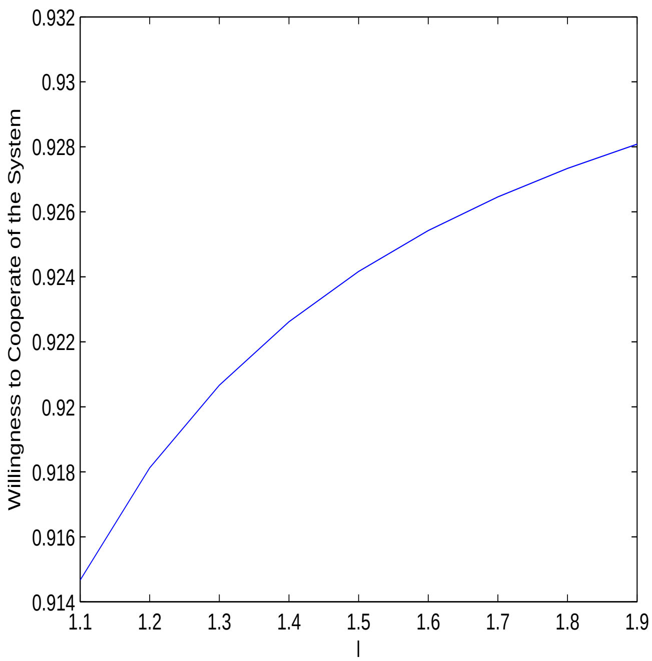

Fig. 4. Impact of increasing l on ϕ

V. Conclusions

While hectic negotiations occur before contracts are awarded and projects start, models that predict how the contractors will respond are lacking in the literature [10]. This paper presented a model based on parameters that can be estimated for predicting how a system will respond to a given set of budgets, performance targets and deadlines. The problem we consider is often studied under the umbrella of a System of Systems (SoS). The underlying principle in our model is that values of some parameters related to the system's behavior (the risk-taking ability agility etc) can be obtained from past interactions and used fruitfully in predicting behavior of the system. We also hope that the Markov chain model we proposed above for studying this problem can be effectively used in the negotiations by the SoS to its own advantage. An important feature of our model is its ability to quantify the intention of the system to take entrepreneurial risk, which can manifest itself in working in a more productive/agile manner as well as work with budgets lower than those proposed by the SoS. We also proved analytical properties of our model that support our intuition. Finally, we have provided a simplified model in the Appendix which can be used when the Markov chain property is not easy to verify.

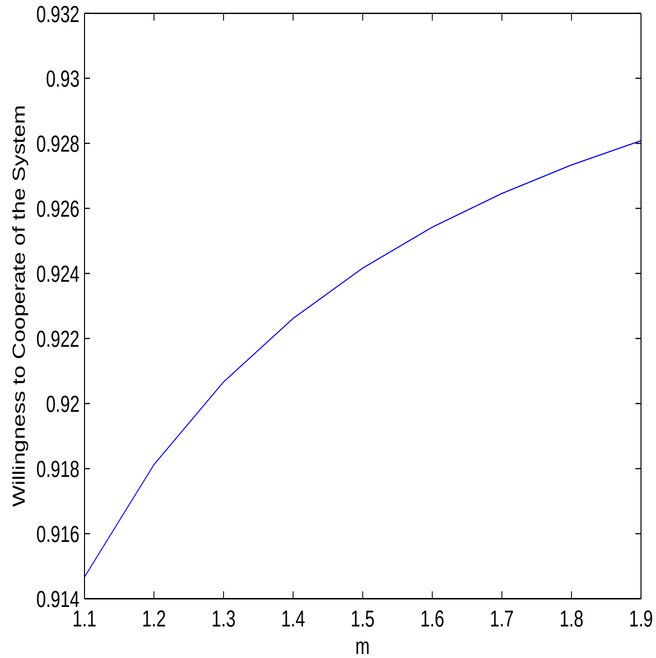

Fig. 5. Impact of increasing m on ϕ

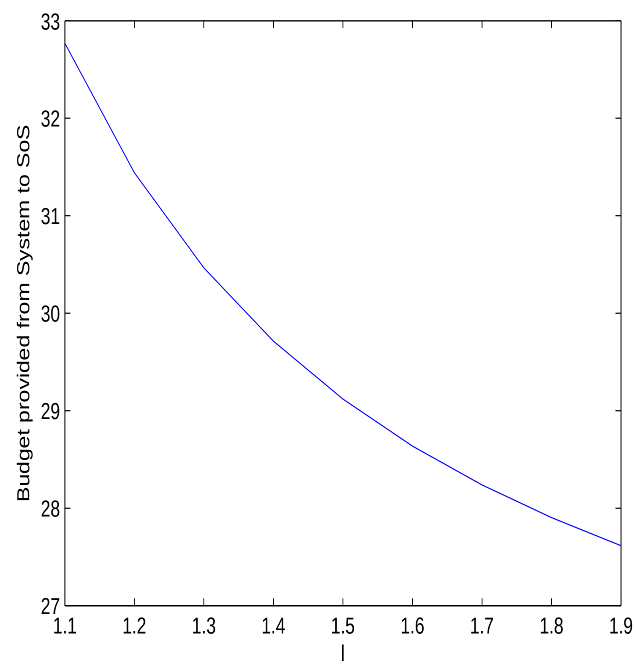

Fig. 6. The impact on the the budget, f_i, with which the system responds, of l

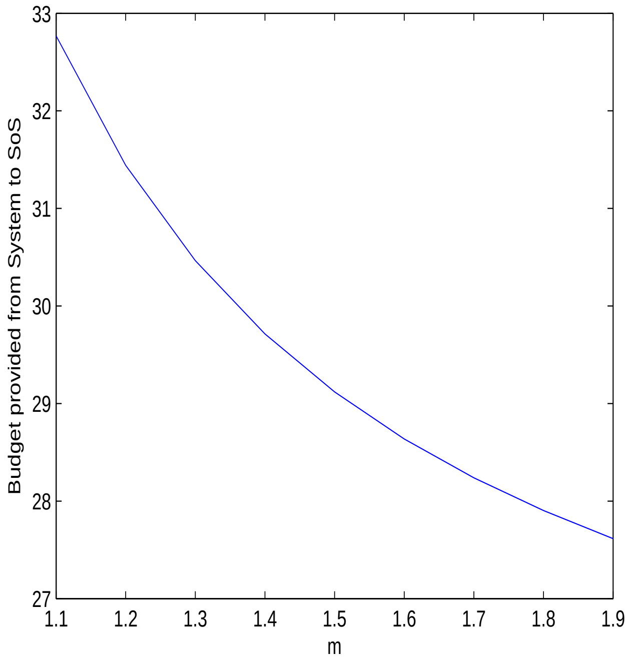

Fig. 7. Impact of m, another agility parameter, on the budget, f_i

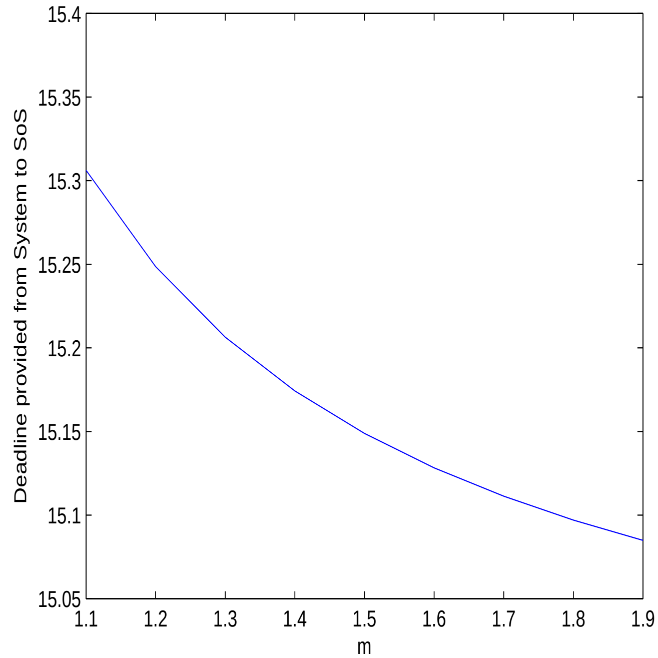

Fig. 9. The deadline agreed to by the system becomes earlier as the agility parameter, l, increases

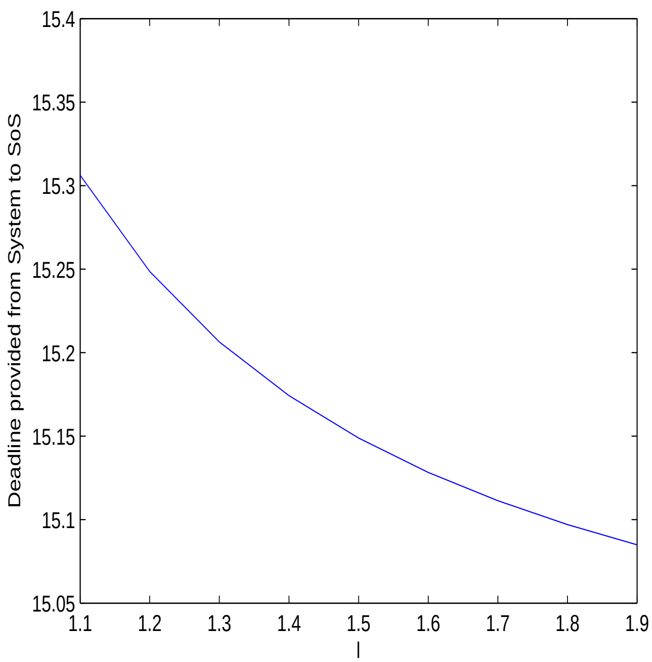

Fig. 8. The deadline agreed to by the system becomes earlier as the agility parameter, l, increases

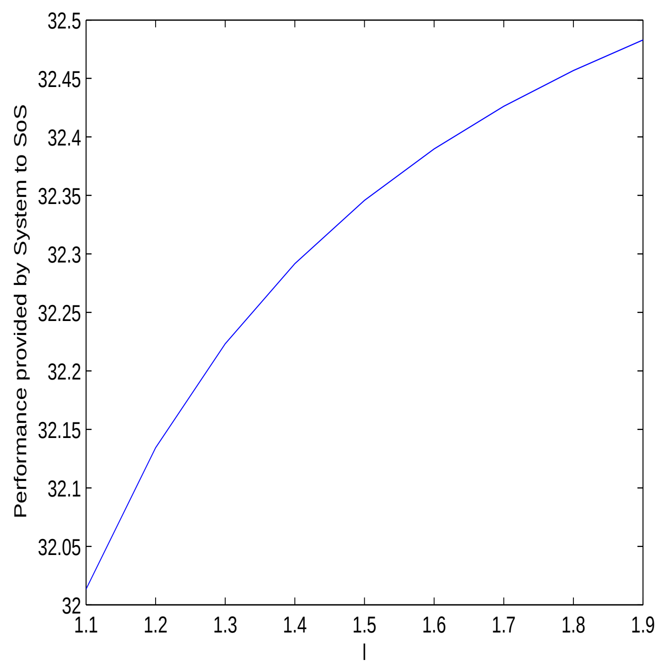

Fig. 10. The graph shows the impact of l on the performance deliverable

Some directions for future research can be envisioned. First, this model could be tested in a re-negotiation modeling discrete-event simulation environment where the SoS returns with a new set of deadlines, budgets, and performance parameters in reaction to the same offered by the system in response to the first offer from the SoS. It may also be interesting to study any potential Nash equilibrium at which the two will eventually settle down. Another potential avenue for future research would be to use simulation modeling, instead of the Markov chain approach employed here, to measure the duration of the project.

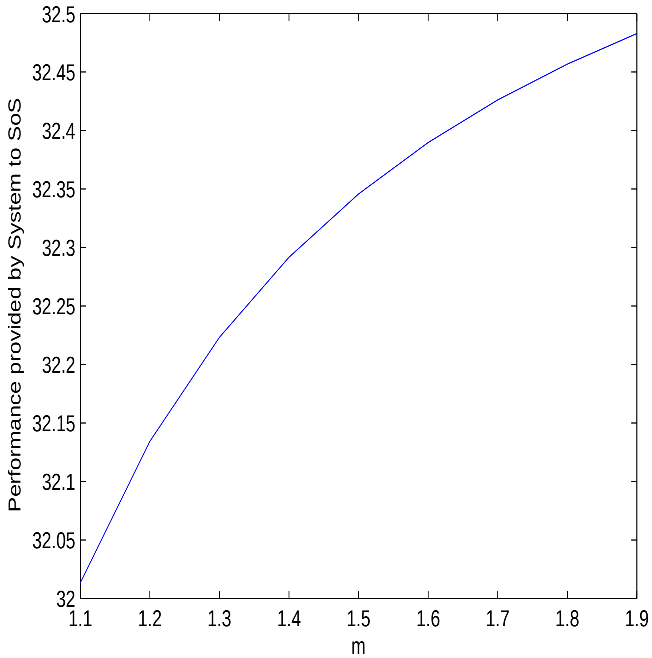

Fig. 11. The graph shows the impact of m on the performance deliverable

Appendix A
Relaxing the Markovian assumption

We now present a short description of how the model proposed above can be used when the transitions in the project stages are not Markovian. When the Markovian assumption is relaxed, it will be necessary to gather data for durations of each phase in the project. Oftentimes, project duration activities are modeled via the triangular distribution [18]. Let X_p denote the transition time from the pth state to the next, and let a_p denote the minimum, b_p denote the maximum, and c_p denote the mode. Then, for a project with three states, we would need to collect data on the triangular distribution for the transition times, i.e., for the triple (a_p, c_p, b_p) for $p = 1, 2$. Then,

$$\phi_i = 1 - \frac{\sum_{p=1}^{2}(a_p + b_p + c_p)}{3(b_1 + b_2)};$$

the above expression stems from the fact that the average value of the time to completion of the two phases, under the triangular distribution, will be $\sum_{p=1}^{2}(a_p + b_p + c_p)/3$ and the maximum value for the same will be $(b_1 + b_2)$.

Appendix B
Projects with more than three states

Although we do not provide a closed form for computing the duration of a project which has more than three states in the Markov chain (as considered above), we explain how the duration can be computed from matrix operations: For any absorbing Markov chain with one absorbing state (project termination), the procedure is analogous to that discussed above. In particular, for four states, if r is defined as the agility parameter for the third state, the transition probability matrix will be:

$$\mathbf{P} = \begin{bmatrix} \frac{k_i}{ln} & 1 - \frac{k_i}{ln} & 0 & 0 \\ 0 & \frac{k_i}{mn} & 1 - \frac{k_i}{mn} & 0 \\ 0 & 0 & \frac{k_i}{rn} & 1 - \frac{k_i}{rn} \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Then, like in the case of three states, one first constructs the $\mathbf{Q}$ matrix by eliminating the absorbing state as follows

$$\mathbf{Q} = \begin{bmatrix} \frac{k_i}{ln} & 1 - \frac{k_i}{ln} & 0 \\ 0 & \frac{k_i}{mn} & 1 - \frac{k_i}{mn} \\ 0 & 0 & \frac{k_i}{rn} \end{bmatrix},$$

and then computes $\mathbf{M}$ via: $\mathbf{M} = \mathbf{I} - \mathbf{Q}$. Defining $\vec{c} = \mathbf{M}^{-1}\vec{w}$, where $\vec{w}$ is a column vector of ones, we have that $c(1)$ will equal τ_i. This process can be extended to any finite number of states.

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