Stochastic Modeling of an Automated Guided Vehicle System With One Vehicle and a Closed-Loop Path

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***Abstract*—The use of automated guided vehicles (AGVs) in material-handling processes of manufacturing facilities and warehouses is becoming increasingly common. A critical drawback of an AGV is its prohibitively high cost. Cost considerations dictate an economic design of AGV systems. This paper presents an analytical model that uses a Markov chain approximation approach to evaluate the performance of the system with respect to costs and the risk associated with it. This model also allows the analytic optimization of the capacity of an AGV in a closed-loop multimachine stochastic system. We present numerical results with the Markov chain model which indicate that our model produces results comparable to a simulation model, but does so in a fraction of the computational time needed by the latter. This advantage of the analytical model becomes more pronounced in the context of optimization of the AGV's capacity which without an analytical approach would require numerous simulation runs at each point in the capacity space.**

***Note to Practitioners*—This paper presents a model for determining the optimal capacity of an automated guided vehicle (AGV) to be purchased by a manufacturer. This paper was motivated by work with manufacturing industries that used AGVs in their operations. The mathematical model we present conducts the performance evaluation of a system with a given number of machines. The performance evaluation done by the model is in terms of: 1) the average inventory in the system; 2) the long-run average cost of operating the system; and 3) the downside risk, which is measured in terms of the probability of leaving a job behind at a workstation by the AGV when it departs. The model can be used to determine the optimal capacity of the AGV needed. It can also help the manager determine whether a trailer needs to be added to an existing AGV. Of particular interest to the managers we interacted with is the issue of downside risk defined above.**

As input parameters, the model requires the knowledge of the distribution of the interarrival time of jobs at each of the workstations. It also needs the mean time taken to travel from one workstation to the next. The model can be easily computerized.

The model we develop is for pick-up AGVs, which primarily pick up jobs and drop them off at a central location in the workplace. With some additional work, our models could be extended to dropoff AGVs. They could also be used for performance evaluation of people movers in amusement parks.

***Index Terms*—Automated guided vehicle (AGV), downside risk, Markov chains, quality-of-service (QoS).**

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I. Introduction

MATERIAL handling is considered to be a critical, albeit non-value-added, activity [25] that can account for 30%–70% of a product's total manufacturing cost [9]. Hence, selecting the appropriate material-handling system assumes importance in the design of production systems. An automated (or automatic) guided vehicle (AGV) possesses advantages, such as flexibility and automation, over conventional material-handling devices, such as forklift trucks and conveyors. It is also known that the AGV is likely to be the material-handling device of "the factory of the future." The use of AGVs in the service industry is also becoming common. For instance, pharmaceutical firms are using AGVs to transport paperwork from one office to another. However, the high cost ($30,000 or more per vehicle) places some restrictions on the number of vehicles that can be purchased and necessitates an economic analysis of AGV systems.

It came to light from interacting with medium-sized industries in Colorado and New York that the capacity of an AGV can be significantly increased, and its effectiveness improved, by *adding a trailer* to the main vehicle [23]. A general conclusion that we were able to draw from our interaction is that for many of these industries, the more pressing problem is not to determine the fleet size, but to determine the capacity of the single AGV that they intend to buy—so that it provides a reasonable degree of quality-of-service (QoS). A common question among managers on the shop floor is: should one buy an additional AGV or a trailer to increase the capacity of the current vehicle, and if yes, what will the impact be of adding a trailer on the performance of the system—both with respect to cost and the QoS?

Electronic manufacturers, large automobile manufacturers, and manufacturing involving hazardous materials require the use of AGVs in larger numbers. From an analytical standpoint, a system with multiple AGVs is oftentimes partitioned into compartments (see Bozer and Srinivasan [3] and Bozer and Park [2]), where *one vehicle* serves a dedicated set of workstations that could either be machines or offices. As a result, in this paper, we focus on the analysis of a closed loop in which there is one vehicle. It turns out that developing a mathematical model for optimization and/or performance evaluation in a stochastic environment—even for a single AGV system with a closed loop—can be quite challenging.

Regardless of the nature of loading, unloading, and demand characteristics, the material-handling system has a significant influence on both of the following: 1) the amount of inventory in the production system and 2) the economic performance of the system. An increase in the capacity of the AGV is likely to reduce the inventory in the system, but the tradeoff here is against the expenses incurred in buying a higher capacity vehicle. Reducing inventory leads to reduced inventory-holding costs and congestion. The cost of an AGV most certainly depends on its capacity. Attaching a trailer to an AGV increases its capacity, and hence variable-capacity AGVs are seen in many real-world systems. Although attaching a trailer can constrain its movements in some ways, buying a new AGV is a considerably more expensive proposition.

Because of the complexity in a stochastic AGV system, analytical models for performance evaluation are not commonly found in the literature. There are, of course, important exceptions, some of which we discuss next. Maxwell and Muckstadt [16] presented an analytical deterministic model to find the minimum number of AGVs required in a given system, which was extended by Leung *et al.* [14] to consider additional vehicle types. Egbelu [6] described four analytical models for a system similar to that of Maxwell and Muckstadt [16]. Tanchoco *et al.* [22] present a model based on the CAN-Q software (Computerized Analysis of Network of Queues, see Solberg [19]) for determining the number of AGVs needed. CAN-Q uses sophisticated queuing theory concepts, but its black-box nature has perhaps prevented further use. Similarly, Wysk *et al.* [27] present a CAN-Q-based analytical model to estimate the number of AGVs in which empty vehicle travel is considered. Bakkalbasi [1] formulates two analytical models; one of his models provides lower and upper bounds of empty traveling time. An analytical model is provided in Mahadevan and Narendran [15] for determination of the number of AGVs in a flexible manufacturing system. Srinivasan *et al.* [20] present a queuing model to determine the throughput capacity of an AGV system. Koo *et al.* [11] use a queuing model to determine the AGV fleet size under a variety of vehicle selection rules. Vis *et al.* [26] develop a model that can be used at an automated container terminal. Chevalier *et al.* [4] address a problem in a system that contains two stations. Johnson and Brandeau [10] present an excellent overview of modeling and design issues pertinent to stochastic material-handling systems. Thonemann and Brandeau [24] use queuing approximations from Gendreau [8] and Powell [17] to study an AGV system in which there is one depot and multiple machines that require material from the depot. Markov chain approximations are popular in uniformization of continuous-time Markov chains [21], diffusion approximations (see [12, Ch. 4]), and financial engineering [5] but have not been used in the analysis of material-handling systems or queuing, to the best of our knowledge.

Contributions of this paper: The capacity of the AGV is an important design issue from a managerial perspective. In this paper, we present for the first time, to the best of our knowledge, a Markov chain approximation model for determining: 1) a number of important performance measures of an AGV system with one vehicle and a closed loop path and 2) the optimal capacity of the AGV. Efficacy of the model is demonstrated with numerical experiments. The latter indicate that its performance is comparable to that of a simulation model; note that simulation models are usually guaranteed to be exact in an *almost sure* sense. Our Markov chain model requires less computation time in comparison to the simulation model, thereby making it useful for optimization purposes (optimization of the vehicle's capacity). Also, we were able to prove that the underlying Markov chain has a special structure which facilitates decomposition of the Markov chain associated with the closed-loop system into a finite number of Markov chains that have a smaller state space. The special structure makes the analysis easier and simplifies the computations considerably because it is easier to handle the smaller Markov chains associated with the subsystems. Furthermore, the model can be used for some distributions in the arrival of jobs at machines that satisfy a property that we identify. Depending on the nature of the system, the pick-up points could either be production machines (jobs) or offices (paperwork), and the dropoff point could be a conveyor belt or the main office. The work of Thonemann and Brandeau [24] is closest to our work in spirit because they also analyze a single vehicle in a closed loop path, but they primarily deal with a "dropoff" system, i.e., a system in which the AGV drops off material at each point and picks up material at a depot. The randomness in their system is in the arrival of jobs to the depot. They do not optimize the AGV's capacity and consider only Poisson arrivals. We focus on a "pick-up AGV," (found in local industries in Colorado) i.e., an AGV that picks up loads at various stations and drops them off at one point. The Markov chain approach enables us to compute some QoS measures, e.g., the probability of the AGV departing from a station leaving x number of jobs stranded, and determine the optimal capacity of the AGV. To the best of our knowledge, this is the first attempt at both of these tasks in the literature.

The rest of this paper is organized as follows. Section II describes the problem. Section III develops the Markov chain model. The performance measures and the optimization model are described in Section IV. Numerical results are presented in Section V. The last section presents some conclusions drawn from our work and some directions for further research in this topic.

II. Problem Description

We first discuss some important features of the problem under consideration. The AGV travels in a fixed circuit from a dropoff point (e.g., conveyor belt) to each machine in a sequence (Machine 1, then Machine 2, and so on until all the machines have been visited) picking up jobs from each of the machines. The AGV then returns to the dropoff point to drop the jobs off and then repeats the circuit. The AGV takes a fixed (deterministic) amount of time to travel from one location to another; this time includes the unloading or loading time. The AGV empties itself completely at the dropoff point. Also, we assume that the route of the AGV is not influenced by whether it is full, although when it is full, it cannot pick up any more jobs in that trip. Jobs arrive at each location with random interarrival times that are independent and identically distributed; the amount of space (output buffer) near the pick-up point (machine) is fixed. In other words, when this buffer is full, the machine stops producing and thus the number of jobs waiting at each machine has an upper limit. Although the amount of space (buffer) at the machine is fixed, it is assumed that the buffer is of a sufficient size that the maximum number of jobs that can be waiting at the machine is rarely reached. Fig. 1 presents a schematic of the system.

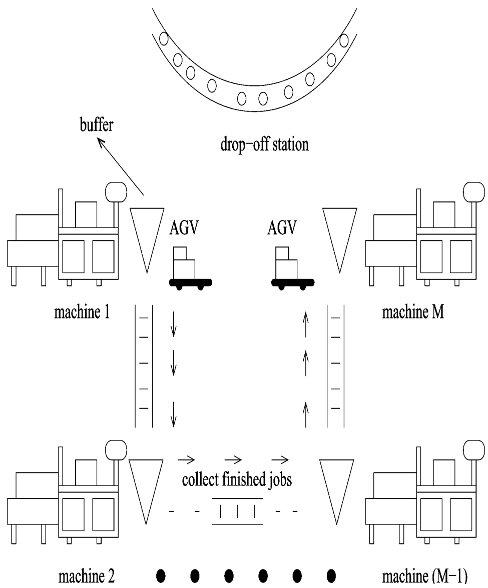

Fig. 1. Schematic view of n-machine system.

The performance metrics that we explore for evaluation and optimization of the system are:

1) average number of jobs waiting at each machine;
2) probability that the number of jobs waiting at a machine *when the AGV departs from the same machine* exceeds a given value;
3) average long-run cost of running the system.

III. Markov Chain Model

A. System State

We introduce a Markov chain model for analyzing the system. Let the set R denote the entire system that contains M machines (or pick-up points). Then

$$R=\{\text{Dropoff point, Machine 1, Machine 2}, \ldots, \text{ Machine } M\}.$$

A subset of R, called $R(n)$, can be defined as follows:

$$R(n) = \{\text{Dropoff point, Machine } n\} \tag{1}$$

for $n = 1, 2, \ldots, M$. The behavior of the above-described system can be modeled with a sequence of Markov chains associated with the following:

$$\{R(1), R(2), \ldots, R(M)\}.$$

The Markov chain associated with $R(n)$ will be referred to as the nth Markov chain. Because each machine will be analyzed independently, we consider M Markov chains, one for each machine. Also, a nice property (Theorem 4.2) of the system is that one can use the invariant distribution (limiting probabilities) of the nth Markov chain to compute the same for the $(n+1)$th Markov chain. This provides for a simple recursive scheme that can be used to determine some important system performance measures associated with the entire system from the invariant distributions.

In order to construct a discrete-time Markov chain, i.e., *uniformize*, we must discretize time. We let Δ denote a positive and small unit of time. We then have the following definition.

1) Definition 3.1: For $\psi > 0$, let $\Delta(\psi, n)$ denote the maximum length of the time interval during which the probability of two or more arrivals of jobs, at the nth machine, is less than ψ, i.e.,

$$\Delta(\psi, n) = \sup\left\{\tau \mid \sum_{i=2}^{\infty} \mathsf{P}[W_n(\tau) = i] < \psi\right\} \tag{2}$$

where $W_n(\tau)$ is the number of arrivals at the nth machine during a time interval of length τ.

Next we introduce some notation.

$\bar{X}(n)$	Maximum number of jobs that can wait at the nth machine.
$T_a(i,j)$	Time spent by the AGV in traveling from machine i to machine j where machine 0 is the dropoff point. This also includes the loading time on machine i when $j \neq 0$ and the unloading time when $j = 0$.
$T_\psi(i,j)$	An integer multiple of $\Delta(\psi, n)$ such that $$T_a(i,j) \approx T_\psi(i,j)\Delta(\psi,n). \tag{3}$$
$p_\psi(j,n)$	Probability that j jobs arrive at the nth machine in a time interval of length $\Delta(\psi, n)$.
$N(n)$	Number of states in the nth Markov chain. (States are defined below in Definition 3.2.)
Z	*Maximum* capacity of the AGV.
β	*Actual* available capacity of the AGV. (It is a random variable for all machines but the first for which it equals Z.)

Let $\tau_K(n)$ denote the time between the Kth and the $(K+1)$th arrival of jobs at machine n. Then, since jobs continually arrive at each machine, for any n

$$\inf\{\tau_K(n) \mid K = 1, 2, \ldots\} = 0. \tag{4}$$

We will observe the system after unit time, i.e., $\Delta(\psi, n)$, following a standard convention in the literature (see [18, p. 435]), and after unit time, a new epoch will be assumed to have begun. Thus, if one specifies $\psi > 0$, the length (time duration) of any epoch in the nth Markov chain will equal $\Delta(\psi, n)$.

The following phenomena will be treated as events for the nth Markov chain: 1) A job arrives at a machine; 2) the AGV departs from the nth machine; and 3) the AGV arrives at the nth machine. An event will signal the beginning of a new epoch. The probability of two or more arrivals in one epoch will be assumed to be negligible for small ψ, and the definition of $\Delta(\psi, n)$ ensures that. Furthermore, we will assume that if a job arrives during an epoch, we will move that event forward in time to coincide with the end of that epoch. Since $\Delta(\psi, n)$ is a small quantity, when ψ is small, this should not pose serious problems. This approximation is necessary to ensure the Markov property for the system we consider. However, numerical results (presented later) will demonstrate that with the approximation we still have reasonably accurate results. Moreover, the approximation allows us to use the powerful framework of Markov chains.

2) Definition 3.2: Specify $\psi > 0$. A state of the nth Markov chain in the mth epoch is defined by the following 4-tuple:

$$X_\psi(n,m) = \langle x(n,m), y(m), s(m), t(m)\rangle \quad (5)$$

where $x(n,m)$ denotes the number of jobs waiting at machine n when the mth epoch begins, $y(m)$ denotes the available capacity in the AGV when the mth epoch begins, and $s(m) = 0$ if the AGV is traveling from the dropoff point to any machine while $s(m) = n$ if the AGV is traveling from the nth machine to the dropoff point when the mth epoch begins. Since the mth epoch could begin while the AGV is traveling either between the dropoff point and the machine n or between the machine n and the dropoff point, we let $t(m)$ denote the number of multiples of $\Delta(\psi,n)$ that have elapsed until the beginning of the mth epoch since the AGV's departure. Clearly, the AGV's departure could either be from the dropoff point or the nth machine.

B. Transition Probabilities

With the above discretization of time, we have the following property. For any small value of $\psi > 0$ and for any n, it is approximately true that

$$\begin{aligned}&\mathsf{P}[X_\psi(n,m+1)|X_\psi(n,m)]\\&= \mathsf{P}[X_\psi(n,m+1)|X_\psi(n,m), X_\psi(n,m-1),\ldots,X_\psi(n,0)].\end{aligned}$$

This implies that for nonzero, but small, ψ, our system can be approximately modeled by a Markov chain. It is to be noted that the Markov chains constructed are essentially parametrized by the non-negative scalar ψ. However, to keep the notation simple, and because ψ is fixed, we will suppress this parameter. Thus, $p_\psi(j,n)$ will be denoted by $p(j,n)$. By the definition, the nth Markov chain is *only* concerned with job arrivals at the nth machine.

The definition of $\Delta(\psi,n)$ requires us to analyze whether the probability of two or more arrivals in an epoch can be ignored in comparison to that of zero or one arrival. We justify the use of a small ψ with the following properties. Consider the following two properties assuming τ to be the duration of an epoch.

Property 1:

$$\lim_{\tau\to 0}\frac{\sum_{k=2}^{\infty}\mathsf{P}[W_n(\tau)=k]}{\mathsf{P}[W_n(\tau)=0]} = 0. \quad (6)$$

Property 2:

$$\lim_{\tau\to 0}\frac{\sum_{k=2}^{\infty}\mathsf{P}[W_n(\tau)=k]}{\mathsf{P}[W_n(\tau)=1]} = 0. \quad (7)$$

When the arrival distribution satisfies these two properties, one can ignore the probability of two or more arrivals in comparison to those of zero arrivals and one arrival as τ tends to zero because the probability of two or more arrivals converges to zero *faster* than either of the other two probabilities. In other words, for a small value of τ, one may practically ignore the event associated with more than two arrivals. With a small ψ, we have for every n a small enough $\Delta(\psi,n)$, i.e., τ in the two properties above. Thus, with a sufficiently small duration of the epoch, one can safely ignore the probability of two or more arrivals for certain distributions. We will prove this when the interarrival time is exponentially distributed and uniformly distributed. Note, however, that even for these distributions, e.g., exponential, our approach remains approximate.

1) Theorem 3.1: If the interarrival time is exponentially distributed with mean $1/\lambda$, Properties 1 and 2 are satisfied.

Proof:

$$\begin{aligned}\lim_{\tau\to 0}\frac{\sum_{k=2}^{\infty}\mathsf{P}[W_n(\tau)=k]}{\mathsf{P}[W_n(\tau)=0]} &= \lim_{\tau\to 0}\frac{1-e^{-\lambda\tau}-\lambda\tau e^{-\lambda\tau}}{e^{-\lambda\tau}}\\&=0\end{aligned}$$

thereby satisfying Property 1. Similarly

$$\begin{aligned}\lim_{\tau\to 0}\frac{\sum_{k=2}^{\infty}\mathsf{P}[W_n(\tau)=k]}{\mathsf{P}[W_n(\tau)=1]} &= \lim_{\tau\to 0}\frac{1-e^{-\lambda\tau}-\lambda\tau e^{-\lambda\tau}}{\lambda\tau e^{-\lambda\tau}}\\&= \lim_{\tau\to 0}\frac{\lambda e^{-\lambda\tau}+\lambda^2\tau e^{-\lambda\tau}-e^{-\lambda\tau}\lambda}{\lambda e^{-\lambda\tau}-\lambda^2\tau e^{-\lambda\tau}}\\&\quad\text{by L'Hospital's rule}\\&=0.\end{aligned}$$

■

2) Theorem 3.2: If the interarrival time is uniformly distributed with $U(0,b)$, Properties 1 and 2 are satisfied.

Proof: For non-Poisson arrivals, to determine the probability distribution of the number of arrivals, one has to compute F_k, the k-fold convolution of the distribution of the interarrival time. If S_k denotes the time of the kth arrival (of job), then for the uniform distribution $U(0,b)$

$$\mathsf{P}[W_n(\tau)=0] = \mathsf{P}[S_1>\tau] = \frac{b-\tau}{b}.$$

Also

$$F_1(\tau) \equiv \mathsf{P}[S_1\le\tau] = \frac{\tau}{b}$$

and

$$F_2(\tau) \equiv \mathsf{P}[S_2\le\tau] = \int_0^{\tau}\int_0^{\tau-x_2}\frac{1}{b}\frac{1}{b}dx_1dx_2 = \frac{\tau^2}{2b^2}.$$

Then

$$\mathsf{P}[W_n(\tau)=1] = F_1(\tau)-F_2(\tau) = \frac{\tau}{b}-\frac{\tau^2}{2b^2}.$$

Then, it follows that:

$$\begin{aligned}\lim_{\tau\to 0}\frac{\sum_{i=2}^{\infty}\mathsf{P}[W_n(\tau)=i]}{\mathsf{P}[W_n(\tau)=0]} &= \lim_{\tau\to 0}\frac{1-\frac{b-\tau}{b}-\frac{\tau}{b}+\frac{\tau^2}{2b^2}}{\frac{b-\tau}{b}}\\&=0.\end{aligned}$$

Property 1 is thus satisfied. Similarly, for Property 2, we have

$$\lim_{\tau\to 0}\frac{\sum_{i=2}^{\infty}\mathsf{P}[W_n(\tau)=i]}{\mathsf{P}[W_n(\tau)=1]} = \lim_{\tau\to 0}\frac{\frac{\tau^2}{2b^2}}{\frac{\tau}{b}-\frac{\tau^2}{2b^2}}$$
$$= \lim_{\tau\to 0}\frac{\frac{\tau}{b^2}}{\frac{1}{b}-\frac{\tau}{b^2}} = 0$$
by L'Hospital's rule.

■

We show that the conditions (6) and (7) hold for the triangular distribution in Appendix I.

For interarrival distributions (of jobs) that satisfy (6) and (7), we can develop expressions for the transition probabilities with a finite but small value of ψ. The transitions are characterized by 12 conditions. For each Markov chain that we consider, we will assume, for the time being, that the available capacity β is known. Subsequently, we will present a result (Theorem 4.2) to compute the distribution of β. The state in the mth epoch will be denoted by $X(n,m)$. The one-step transition probability will be denoted by $\mathsf{P}[X(n,m+1)|X(n,m)]$.

Fig. 2 presents a pictorial representation of the underlying Markov chain in our model.

The available capacity of the AGV, β, equals Z when it arrives at the first machine and is a random variable for all other machines. Then, $s(m)\in\{0,n\}$, $t(m)\in\{0,1\ldots,T(0,n)\}$ if $s(m)=0$, $t(m)\in\{0,1\ldots,T(n,0)\}$ if $s(m)=n$ (*see 3.2*), $X(n,m)\in\{0,1,\ldots\bar{X}(n)\}$ and $y(m)\in\{0,1,\ldots,\beta\}$. The transition probabilities for this Markov chain will now be defined for 12 conditions (cases). Cases 1–6 (7–12) are associated with the AGV traveling from the dropoff point to a machine (from a machine to the dropoff point). We now describe all the cases in detail.

Consider the following scenario: the AGV travels from the dropoff point to machine n during the mth epoch, the arrival of the AGV at machine n does not occur by the end of the epoch, and the buffer at machine n is not full. Then, if no job arrives during the mth epoch, we have the following.

Case 1: If $t(m)<T(0,n)-1$, $x(n,m)<\bar{X}(n)$, $s(m)=0$, $x(n,m+1)=x(n,m)$, $y(m+1)=y(m)$, $s(m+1)=s(m)$, and $t(m+1)=t(m)+1$, then

$$\mathsf{P}[X(n,m+1)|X(n,m)]=p(0,n).$$

Since no job arrival occurs in "unit time," the above transition probability is equal to the probability of no job arrival. Similarly, if a job arrival does occur, we have the following.

Case 2: If $t(m)<T(0,n)-1$, $x(n,m)<\bar{X}(n)$, $s(m)=0$, $x(n,m+1)=x(n,m)+1$, $y(m+1)=y(m)$, $s(m+1)=s(m)$, and $t(m+1)=t(m)+1$, then

$$\mathsf{P}[X(n,m+1)|X(n,m)]=1-p(0,n).$$

Since job arrival occurs in "unit time," the above transition probability is equal to the probability of job arrival.

Consider the following scenario: the AGV travels from the dropoff point to the machine n during the mth epoch, the arrival of the AGV at machine n does not occur by the end of the epoch and the buffer at machine n is full. Then, we have the following.

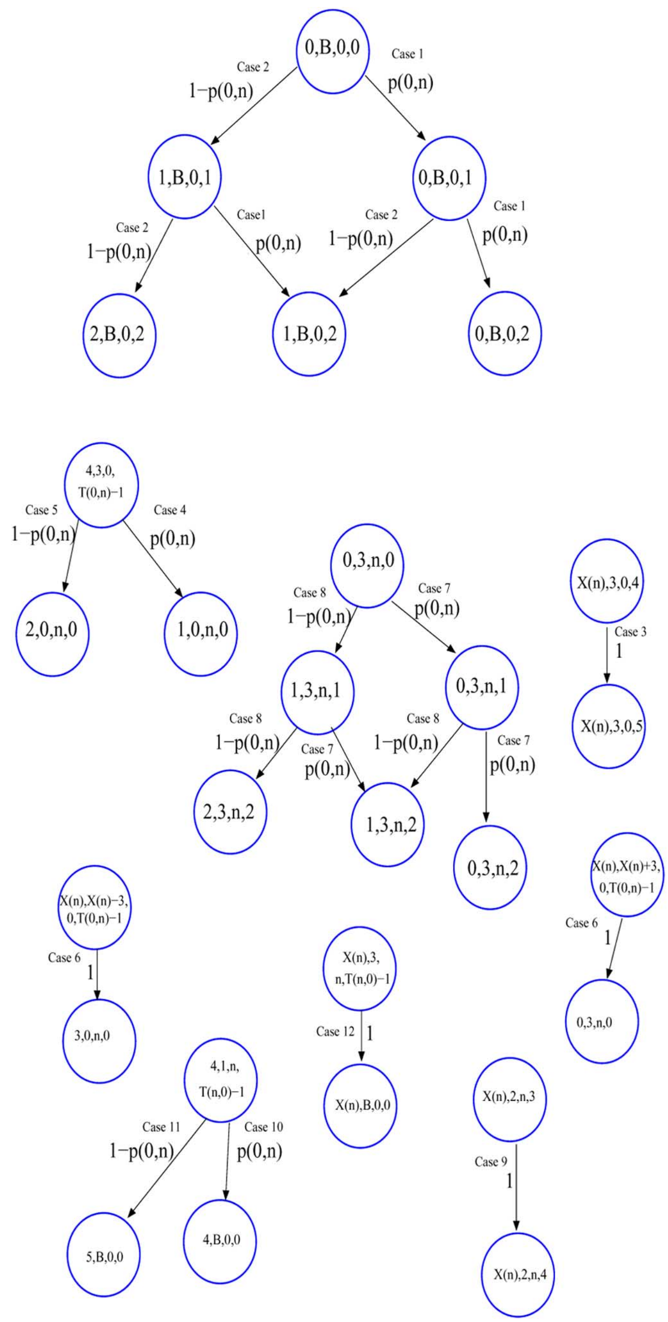

Fig. 2. Markov chain underlying our model. B in figure stands for β.

Case 3: If $t(m)<T(0,n)-1$, $x(n,m)=\bar{X}(n)$, $s(m)=0$, $x(n,m+1)=\bar{X}(n)$, $y(m+1)=y(m)$, $s(m+1)=s(m)$, and $t(m+1)=t(m)+1$, then

$$\mathsf{P}[X(n,m+1)|X(n,m)]=1.$$

Since the buffer at machine n is full, we do not take job arrivals into consideration.

Consider the following scenario: the AGV travels from the dropoff point to the machine n during the mth epoch, it arrives at machine n by the end of the epoch, and the buffer at machine n is not full. Then, if no job arrives during the current epoch, we have Case 4, and if a job arrives we have Case 5.

Case 4: If $t(m) = T(0,n) - 1$, $x(n,m) < \bar{X}(n)$, $s(m) = 0$, $x(n,m+1) = \max[0, x(n,m) - y(m)]$, $y(m+1) = \max[0, y(m) - x(n,m)]$, $s(m+1) = n$, and $t(m+1) = 0$, then

$$\mathsf{P}[X(n,m+1)|X(n,m)] = p(0,n).$$

Case 5: If $t(m) = T(0,n) - 1$, $x(n,m) < \bar{X}(n)$, $s(m) = 0$, $x(n,m+1) = \max[0, x(n,m) + 1 - y(m)]$, $y(m+1) = \max[0, y(m) - x(n,m) - 1]$, $s(m+1) = n$, and $t(m+1) = 0$, then

$$\mathsf{P}[X(n,m+1)|X(n,m)] = 1 - p(0,n).$$

Consider the scenario in which the AGV travels from the dropoff point to machine n during the mth epoch, arrives at machine n by the end of the mth epoch, and the buffer at machine n is full. Then, we have the following.

Case 6: If $t(m) = T(0,n) - 1$, $x(n,m) = \bar{X}(n)$, $s(m) = 0$, $x(n,m+1) = \max[0, \bar{X}(n) - y(m)]$, $y(m+1) = \max[0, y(m) - \bar{X}(n)]$, $s(m+1) = n$, and $t(m+1) = 0$, then

$$\mathsf{P}[X(n,m+1)|X(n,m)] = 1.$$

Consider the scenario in which the AGV travels from machine n to the dropoff point during the mth epoch, the arrival to the dropoff point does not occur by the end of the epoch and the buffer at machine n is not full. Then, if no job arrives during the mth epoch, we have Case 7, and if a job does arrive we have Case 8.

Case 7: If $t(m) < T(n,0) - 1$, $x(n,m) < \bar{X}(n)$, $s(m) = n$, $x(n,m+1) = x(n,m)$, $y(m+1) = y(m)$, $s(m+1) = s(m)$, and $t(m+1) = t(m) + 1$, then

$$\mathsf{P}[X(n,m+1)|X(n,m)] = p(0,n).$$

Case 8: If $t(m) < T(n,0) - 1$, $x(n,m) < \bar{X}(n)$, $s(m) = n$, $x(n,m+1) = x(n,m) + 1$, $y(m+1) = y(m)$, $s(m+1) = s(m)$, and $t(m+1) = t(m) + 1$, then

$$\mathsf{P}[X(n,m+1)|X(n,m)] = 1 - p(0,n).$$

Consider the scenario in which the AGV travels from machine n to the dropoff point during the mth epoch, the arrival to the dropoff point does not occur by the end of the mth epoch and the buffer at machine n is full. Then, we have the following.

Case 9: If $t(m) < T(n,0) - 1$, $x(n,m) = \bar{X}(n)$, $s(m) = n$, $x(n,m+1) = \bar{X}(n)$, $y(m+1) = y(m)$, $s(m+1) = s(m)$, and $t(m+1) = t(m) + 1$, then

$$\mathsf{P}[X(n,m+1)|X(n,m)] = 1.$$

Consider the scenario in which the AGV travels from machine n to the dropoff point during the mth epoch, it arrives at the dropoff point by the end of the epoch, and the buffer at machine n is not full. Then, if no job arrives during the mth epoch, we have Case 10. If a job arrival occurs during the mth epoch, we have Case 11.

Case 10: If $t(m) = T(n,0) - 1$, $x(n,m) < \bar{X}(n)$, $s(m) = n$, $x(n,m+1) = x(n,m)$, $y(m+1) = \beta$, $s(m+1) = 0$, and $t(m+1) = 0$, then

$$\mathsf{P}[X(n,m+1)|X(n,m)] = p(0,n).$$

Case 11: If $t(m) = T(n,0) - 1$, $x(n,m) < \bar{X}(n)$, $s(m) = n$, $x(n,m+1) = x(n,m) + 1$, $y(m+1) = \beta$, $s(m+1) = 0$, and $t(m+1) = 0$, then

$$\mathsf{P}[X(n,m+1)|X(n,m)] = 1 - p(0,n).$$

Finally, consider the scenario in which the AGV travels from machine n to the dropoff point during the mth epoch, it arrives at the dropoff point by the end of the epoch, and the buffer at machine n is full. Then, we have the following.

Case 12: If $t(m) = T(n,0) - 1$, $x(n,m) = \bar{X}(n)$, $s(m) = n$, $x(n,m+1) = \bar{X}(n)$, $y(m+1) = \beta$, $s(m+1) = 0$, and $t(m+1) = 0$, then

$$\mathsf{P}[X(n,m+1)|X(n,m)] = 1.$$

IV. Performance Measures and Optimization

From Definition 3.2, the number of states in the nth Markov chain, i.e., $N(n)$, can be computed as

$$N(n) = (\bar{X}(n) + 1)(Z + 1)(T_\psi(n,0) + T_\psi(0,n)).$$

The following well-known result allows us to determine the invariant distribution (limiting probabilities) of the underlying Markov chains.

1) Theorem 4.1: Let $\mathbf{P}$ denote the one-step transition probability matrix of a Markov chain. If the matrix is aperiodic and irreducible, and if $\vec{U}$, whose ith element is denoted by U_i, denotes a column vector of size $N(n)$, then solving the following system of linear equations yields the limiting probabilities of the Markov chain:

$$\vec{U}^T \mathbf{P} = \vec{U}, \text{and} \sum_i U_i = 1.$$

From the definition of our transition probabilities, it is not hard to show that each state is positive recurrent and aperiodic and that there is a single communicating class of states. Then, the above result holds and we may use it to compute the limiting probabilities.

We next define a function, G, that assigns an integer value in the set $\{1, 2, \ldots, N(n)\}$ to each state in the nth Markov chain

$$\begin{aligned} G(x(n), y, s, t) &\equiv [x(n)(Z+1)T(0,n) + yT(0,n) + t + 1] \times \left(1 - \frac{s}{n}\right) \\ &\quad + [(\bar{X}(n) + 1)(Z+1)T(0,n) \\ &\qquad + x(n)(Z+1)T(n,0) + yT(n,0) + t + 1]\frac{s}{n} \end{aligned}$$

where $\langle x(n), y, s, t \rangle$ denotes the state in the nth Markov chain at a given epoch, m. Note that m, the epoch index, is suppressed here from the notation of (5) to increase clarity. Our transition probabilities for the nth Markov chain were computed under the

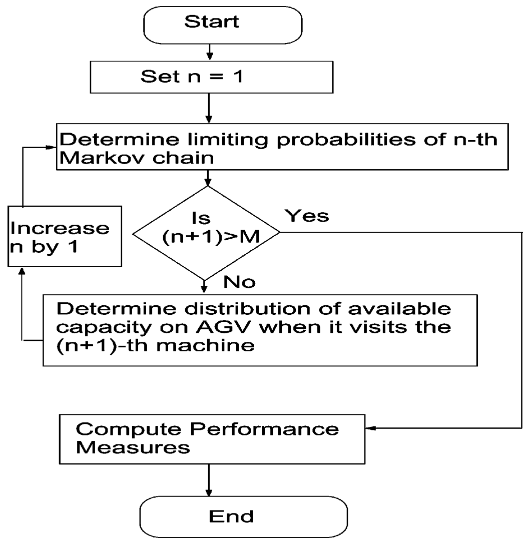

Fig. 3. Computational scheme based on Theorem 4.2.

assumption that capacity of the AGV when it arrives at the nth machine is known (β). We now need to compute the transition probabilities of the entire system—the transition probabilities that we need for our performance evaluation model. We will provide a result, Theorem 4.2, to this end. The main idea underlying the result is as follows. The transition probabilities of the nth Markov chain yield the distribution of the AGV's available capacity of the $(n+1)$th Markov chain. Because the AGV starts empty, for the first Markov chain, the available capacity is known to be Z. From that point onwards, one can compute the transition probabilities in a recursive style. The computational scheme based on the next result is depicted in a flowchart in Fig. 3.

2) Theorem 4.2: Let the limiting probability of state S in the nth Markov chain be denoted by $\pi(G(S), n)$. Let γ_n denote the available capacity of the AGV when it arrives at the nth machine, and further let $L(G(S), n, \beta)$ denote the limiting probability for the state S in the nth Markov chain when $\gamma_n = \beta$. Then, we have that

$$\pi(G(S), 1) = L(G(S), 1, Z), \text{ for } G(S) \in \{1, 2, \ldots, N(1)\} \tag{8}$$

and

$$\pi(G(S), n) = \sum_{\beta=0}^{Z} \mathsf{P}[\gamma_n = \beta] L(G(S), n, \beta) \tag{9}$$

for $G(S) \in \{1, 2, \ldots, N(n)\}$, $n = 2, 3, \ldots, M$, where

$$\mathsf{P}[\gamma_n = \beta] = \frac{\sum_{k \in \bar{V}_1} \pi(k, n-1)}{\sum_{k \in \bar{V}_2} \pi(k, n-1)}, \text{ for } n = 2, 3 \ldots, M \tag{10}$$

in which $\bar{V}_1$ and $\bar{V}_2$ are defined as follows:

$$\bar{V}_1 = \{k | k = G(x, \beta, (n-1), 0), x = 0, 1, \ldots, \bar{X}(n-1)\}$$

and

$$\bar{V}_2 = \{k | k = G(x, \gamma_n, (n-1), 0), x = 0, 1, \ldots, \bar{X}(n-1) \quad \gamma_n = 0, 1, \ldots, Z\}.$$

Proof: For the proof, we need to justify (8)–(10). Equation (8) follows from the fact that the AGV empties itself at the dropoff point (see Fig. 1). Hence, when it comes to Machine 1, the available capacity is not a random variable but a constant equal to Z, which is the maximum capacity of the AGV. Equation (9) follows from the fact that when the AGV arrives at stations numbered 2 through M, its capacity is a random variable whose distribution has to be calculated. The distribution is given in (10). For the last statement, we argue as follows: When the AGV arrives at the nth station, for $n = 2, 3, \ldots, M$, its capacity is the available capacity of the AGV when it leaves the $(n-1)$th station. The distribution for the available capacity on the AGV when it leaves the $(n-1)$th station can be computed from the limiting probabilities associated with the $(n-1)$th Markov chain of those states in which the AGV has departed the $(n-1)$th machine and is on the way to the nth machine. (Note that $\bar{V}_1$ denotes the set of states in the $(n-1)$th Markov chain in which the AGV has departed the $(n-1)$th machine and its available capacity is β. Similarly, $\bar{V}_2$ denotes the set of states in the $(n-1)$th Markov chain in which the AGV has departed from the $(n-1)$th machine and its available capacity assumes all possible values.) ■

Exploiting the transition probabilities, one can derive expressions for some useful performance measures of the system. They are considered next.

3) Average Inventory at Each Machine: If $\mathsf{E}[W(n)]$ denotes the average number of jobs waiting at the nth machine, then

$$\mathsf{E}[w(n)] = \sum_{\chi=0, K \in V_\chi}^{\chi=\bar{x}(n)} \chi \pi(k, n) \tag{11}$$

where

$$V_\chi = \{k | k = G(\chi, y, s, t), y = 0, 1, \ldots, s = 0, n \quad t = 0, 1, \ldots, q\},$$

and

$$q = \begin{cases} T(0, n), & \text{if } s = 0 \\ T(n, 0), & \text{if } s = n \end{cases}.$$

4) QoS Performance Measure Associated With Downside Risk: The probability that the number of jobs waiting at the nth machine, denoted by $D(n)$, when the AGV departs from the nth machine exceeds θ can be calculated as follows:

$$\mathsf{P}[d(n) \geq \theta] = \frac{\sum_{k \in V_\theta} \pi(k, n)}{\sum_{k \in v_0} \pi(k, n)} \tag{12}$$

where

$$V_\theta = \{k | k = G(x, y, n, 0), x = \theta, (\theta+1), \ldots, \bar{X}(n), \quad y = 0, 1, \ldots, Z\}.$$

5) Average Long-Run Cost: The average long-run cost of running the system is composed of two elements, which are: the holding cost of the jobs near the machine and the cost of operating the AGV. To calculate the holding cost, we will need the average number of jobs waiting at each machine. Then, the average cost for operating a system described in this paper, with

M machines to be served by an AGV whose capacity is Z, is given by

$$C = c_1 Z + c_2 \sum_{n=1}^{M} \mathsf{E}[W(n)] + c_3 \tag{13}$$

where c_1 and c_2 are constants representing the operating cost per unit time of an AGV having unit capacity and the holding cost per a unit time for one job, respectively; c_3 denotes the fixed cost of the vehicle

In the uniformization procedure, the probability of two or more arrivals in one epoch is neglected. Thus, a few arrivals are unaccounted for. This causes a *reduction* in the values of both performance measures, i.e., $\mathsf{E}[W(n)]$ and $\mathsf{P}[D(n) \geq \theta]$, for the first machine. This error affects the capacity distribution and results in a *reduced* available capacity for the AGV when it visits the subsequent machines. This error in the capacity distribution *partly negates* the error introduced by uniformization. Hence, at all machines but the first, where there is no error in estimating the capacity, the overall error in the performance-measure values is low. At the first machine, the capacity is equal to the maximum capacity, and so this reduction does not occur, thereby producing a higher error. This is reflected in the numerical results presented later.

Also note that one has to find the limiting probabilities of M different Markov chains, one chain associated with a specific machine in the system. However, to find the limiting probability of the specific state in a Markov chain associated with machine i, where $i = 2, \ldots, M$, one has to find the limiting probabilities of the $(Z+1)$ different Markov chains associated with machine i (see Theorem 4.2), since the available capacity of the AGV can assume any value between zero and Z when it arrives at any machine but the first machine. (Note that since the available capacity of the AGV is Z when it arrives at the first machine, the limiting probabilities of the Markov chain associated with the first machine can be found without the use of Theorem 4.2.) Hence, the computational work associated with any given machine but the first constitutes of the computational work of calculating the limiting probabilities of $(Z+1)$ Markov chains plus the limiting probabilities of the Markov chain associated with the given machine via Theorem 4.2. Therefore, the complexity of the Markov chain model is a first-order polynomial in the number of machines M. *The advantage of our computational scheme is that the state space collapses for each Markov chain, thereby making the computation of the limiting probabilities feasible.*

6) AGV Capacity Optimization: We are interested in optimizing the AGV's capacity with respect to the cost of operating the system. The optimization problem considered in this paper is to determine Z to minimize the expression in (13). Other ways for optimization could be considered depending on what the manager desires. One example is to

$$\text{minimize } Z \text{such that } \mathsf{P}[D(n) \geq \theta] \leq \bar{\lambda}, \quad n = 1, 2, \ldots, M$$

where θ and $\bar{\lambda} \in (0, 1)$ are set by managerial policy.

Optimization is performed by an exhaustive enumeration of the AGV's maximum capacity variable, Z. Since the state space of this discrete optimization problem, which has a single decision variable, is very small, an exhaustive enumeration is feasible.

V. Computational Results

Simulation is by far the most extensively used tool for performance evaluation of AGV systems, because it provides us with very *accurate* estimates of performance measures. As a result, it is essential that we compare our results to those obtained from a simulation model.

A. Simulation Model

Consider a probability space $(\Omega, \mathcal{F}, P)$, where Ω denotes the (universal) set of all possible round trips of the AGV, $\mathcal{F}$ denotes the sigma field of subsets of Ω, and P denotes a probability measure on $(\Omega, \mathcal{F})$. Using a discrete-event simulator, it is possible to generate random samples $\omega^1, \omega^2, \ldots, \omega^k$ from the measurable space. The samples can then be used to estimate values of all the performance measures derived in the previous section. Let $\alpha(n, \omega^i, t)$ denote the number of jobs waiting near the nth machine at time t in the simulation sample ω^i. Then, from the strong law of large numbers, with probability 1

$$\mathsf{E}[W(n)] = \lim_{k\to\infty} \frac{1}{k} \sum_{i=1}^{k} \lim_{\tau\to\infty} \frac{\int_0^\tau \alpha(n, \omega^i, t)dt}{\tau}. \tag{14}$$

Also, with probability 1

$$\mathsf{P}[D(n) \geq \theta] = \lim_{k\to\infty} \frac{1}{k} \sum_{i=1}^{k} \lim_{l\to\infty} \frac{\Gamma(n, \omega^i, \theta, l)}{l} \tag{15}$$

where $\Gamma(n, \omega^i, \theta, l)$ denotes the number of occasions in which the number of jobs left behind at machine n (i.e., jobs not picked up by the AGV because it is full) equals or exceeds θ in l visits to the machine in the simulation sample ω^i.

B. Performance Tests for Markov Model

We conducted numerical experiments with our model to determine the practicality of the approach and to benchmark its performance with a simulation model that is guaranteed to perform well but is considerably slower. The error of our model with respect to the simulation estimate is defined by

$$\text{Error } (\%) = \frac{|\kappa_{MC} - \kappa_{\text{SIM}}|}{\kappa_{\text{SIM}}} \times 100$$

where κ_{MC} denotes the estimate from the Markov chain model and κ_{SIM} denotes the same from the simulation model. We present results on two-machine and five-machine systems, along with a simple example to illustrate our methodology.

Tables I and II define the parameters for the systems with two machines, and Tables XI and XII define the parameters for the systems with five machines studied. In Table I, λ_i denotes the rate of arrival of jobs at machine i. We used the exponential and gamma to model the distribution of the interarrival time of jobs. The performance metrics of the systems with two machines and five machines are shown in Tables III–VIII and Tables XIII–XX, respectively. These tables also show the simulation estimates and the corresponding error values which are calculated as defined above. Convergence was achieved with

TABLE I
SYSTEM (SYS.) PARAMETERS. VALUE OF ψ IS 0.05 FOR EACH SYSTEM AND EACH SYSTEM HAS TWO MACHINES

Sys.	$\lambda_1 = \lambda_2$	$\Delta(\psi, n)$	Z	$X_1 = X_2$
1	1.9	0.187	2	3
2	1.9	0.187	2	4
3	1.9	0.187	2	3
4	2.0	0.177	2	3
5	2.0	0.177	2	4
6	2.0	0.177	2	3
7	1.5	0.2369	1	2
8	1.5	0.2369	2	3
9	1.5	0.2369	2	5
10	1.5	0.2369	2	4
11	1.5	0.2369	2	2
12	1.5	0.2369	2	3
13	1.5	0.2369	2	4
14	1.5	0.2369	2	3
15	1.5	0.2369	2	3
16	1.5	0.2369	2	5
17	1.5	0.2369	2	4
18	1.5	0.2369	2	5
19	1.5	0.2369	2	3

TABLE II
SYSTEM (SYS.) PARAMETERS (CONTINUED)

Sys.	$T(0,1)$	$T(1,0)$	$T(0,2)$	$T(2,0)$
1	5	10	9	6
2	5	10	9	6
3	6	14	14	6
4	5	10	9	6
5	5	10	9	6
6	6	14	14	6
7	6	12	12	6
8	6	12	12	6
9	3	9	7	5
10	3	9	7	5
11	9	15	12	12
12	3	12	7	8
13	3	12	7	8
14	6	14	10	10
15	6	11	12	5
16	2	10	8	4
17	2	10	8	4
18	3	7	5	5
19	5	14	13	6

TABLE III
AVERAGE NUMBER OF JOBS WAITING AT MACHINE 1 FOR POISSON ARRIVALS

	$\mathsf{E}[W(1)]$		
Sys.	Markov Chain	Simulation Model	Error %
1	2.318839	2.495585	7.0823
2	3.312900	3.569621	7.1918
3	2.497668	2.686799	7.0393
4	2.318839	2.568639	9.725
5	3.312900	3.562747	7.0128
6	2.497668	2.680788	6.8308
7	1.81151	1.90725	5.0198
8	2.439622	2.648299	7.8797
9	4.088194	4.388564	6.8444
10	3.094501	3.395249	8.8579
11	1.582405	1.71601	7.7858
12	2.311649	2.52254	8.1684
13	3.310392	3.475389	4.7476
14	2.496057	2.617879	4.6535
15	2.402838	2.582303	6.9498
16	4.084067	4.248308	3.866
17	3.090603	3.307955	6.5706
18	3.780827	4.066098	7.0158
19	2.468618	2.627716	6.0546

TABLE IV
AVERAGE NUMBER OF JOBS WAITING AT MACHINE 2 FOR POISSON ARRIVALS

	$\mathsf{E}[W(2)]$		
Sys.	Markov Chain	Simulation Model	Error %
1	2.998591	2.958283	1.363
2	3.999898	3.994344	0.139
3	2.999856	2.996714	0.105
4	2.998591	2.995783	0.094
5	3.999898	3.994419	0.137
6	2.999856	2.996866	0.1
7	2.000003	1.997368	0.132
8	2.999654	2.995505	0.139
9	4.999773	4.987181	0.252
10	3.998825	3.990671	0.204
11	1.999635	1.980528	0.965
12	2.998553	2.94297	1.889
13	3.999894	3.907871	2.355
14	2.999851	2.956217	1.476
15	2.99944	2.963889	1.199
16	4.999762	4.879877	2.457
17	3.998782	3.920698	1.992
18	4.997131	4.854076	2.947
19	2.999771	2.93881	2.074

TABLE V
AVERAGE NUMBER OF JOBS WAITING AT MACHINE 1 FOR GAMMA-DISTRIBUTED INTERARRIVAL TIME

	$\mathsf{E}[W(1)]$		
Sys.	Markov Chain	Simulation Model	Error %
1	2.783362	2.947502	5.5683
2	3.783362	3.945535	4.1103
3	2.837522	2.958426	4.0868
4	2.788315	2.943774	5.28
5	3.788315	3.9957	5.19
6	2.8412	2.9549	3.84
7	1.92627	1.987428	3.07
8	2.77881	2.9366	5.37
9	4.6682	4.8994	4.72
10	3.668215	3.904053	6.04
11	1.834107	1.953312	6.10
12	2.734572	2.92587	6.5382
13	3.734572	3.922349	4.7874
14	2.800929	2.941225	4.77
15	2.765799	2.932094	5.67
16	4.668215	4.8995	4.72
17	3.6682	3.903247	6.02
18	4.601851	4.880455	5.7086
19	2.790452	2.940642	5.10

1000 trips per replication and we used ten replications. The mean estimate was assumed to have converged when it remained within 0.05% in the next iteration. The mean reported is an average over all replications. The standard deviation was no more than 0.1% of the mean in each case.

1) 2-Machine Systems: Tables III and IV provide $\mathsf{E}[W(n)]$ for Poisson arrivals for $n = 1$ and $n = 2$, respectively. Similarly, Tables V and VI show the corresponding values for a gamma-distributed interarrival time. In these systems, i.e., in the systems with two machines, for the gamma distribution $G(l, \lambda)$ we used the same arrival rate, but the parameter l was set to eight. Tables VII and VIII show the values of $\mathsf{P}[D(n) \geq 2]$ with Poisson arrivals for $n = 1$ and $n = 2$, respectively; Tables IX and X show the corresponding values for a gamma-distributed interarrival time.

TABLE VI
AVERAGE NUMBER OF JOBS WAITING AT MACHINE 2 FOR GAMMA-DISTRIBUTED INTERARRIVAL TIME

	$\mathsf{E}[W(2)]$		
Sys.	Markov Chain	Simulation Model	Error %
1	3.0	2.997188	0.09
2	4.0	3.995275	0.11
3	3	2.99701	0.09
4	3.0	2.997543	0.08
5	4	3.941571	1.4
6	3.0	2.9977	0.07
7	2.0	1.998375	0.08
8	3.0	2.9963	0.12
9	5.0	4.9899	0.20
10	4.0	3.9934	0.16
11	2.0	1.9983	0.08
12	3.0	2.9962	0.1249
13	4.0	3.99339	0.16
14	3.0	2.996312	0.12
15	3.0	2.9963	0.12
16	5.0	4.989429	0.21
17	4.0	3.993118	0.17
18	5.0	4.9899	0.20
19	3.0	2.9963	0.12

TABLE VII
PROBABILITY THAT AGV LEAVES TWO OR MORE JOBS BEHIND FOR POISSON ARRIVALS

	$\mathsf{P}[D(1) \geq 2]$		
Sys.	Markov Chain	Simulation Model	Error %
1	0	0	0
2	0.959948	0.966387	0.6663
3	0	0	0
4	0	0	0
5	0.959948	0.965782	0.6041
6	0	0	0
7	0	0	0
8	0	0	0
9	0.972649	0.973295	0.0664
10	0.890340	0.908807	2.032
11	0	0	0
12	0	0	0
13	0.959236	0.932143	2.906
14	0	0	0
15	0	0	0
16	0.972046	0.922222	5.403
17	0.888749	0.844444	5.247
18	0.91817	0.9	2.019
19	0	0	0

TABLE VIII
PROBABILITY THAT AGV LEAVES TWO OR MORE JOBS BEHIND FOR POISSON ARRIVALS

	$\mathsf{P}[D(2) \geq 2]$		
Sys.	Markov Chain	Simulation Model	Error %
1	0.999788	0.998876	0.091
2	0.999999	0.999438	0.056
3	0.999993	0.999251	0.074
4	0.999788	0.998936	0.085
5	0.999999	0.998936	0.106
6	0.999993	0.999291	0.07
7	0.999997	0.999573	0.042
8	0.999973	0.999145	0.083
9	0.999998	0.998011	0.199
10	0.999960	0.998011	0.195
11	0.997777	0.994118	0.368
12	0.999779	0.978571	2.167
13	0.999999	0.978571	2.19
14	0.999943	0.990476	0.956
15	0.999944	0.992	0.801
16	0.999998	0.968571	3.245
17	0.999958	0.968571	3.241
18	0.999949	0.97619	2.434
19	0.999986	0.977273	2.324

TABLE IX
PROBABILITY THAT AGV LEAVES TWO OR MORE JOBS BEHIND FOR GAMMA-DISTRIBUTED INTERARRIVAL TIME

	$\mathsf{P}[D(1) \geq 2]$		
Sys.	Markov Chain	Simulation Model	Error %
1	0	0	0
2	1	0.99901	0.099098
3	0	0	0
4	0	0	0
5	1	1	0
6	0	0	0
7	0	0	0
8	0	0	0
9	1	0.994059	0.597650642
10	0.999999	0.994059	0.597550
11	0	0	0
12	0	0	0
13	1	0.992593	0.746227306
14	0	0	0
15	0	0	0
16	1	0.990099	1.00000101
17	0.999999	0.990099	0.99990001
18	1	0.990099	1.00000101
19	0	0	0

2) Example: Consider System 5, whose parameters are given in Tables I and II. In this system, jobs arrive at the rate of 2 per unit time at each machine. We choose $\psi = 0.05$. Here, $\Delta(0.05, n) = 0.177$ for $n = 1, 2$. The AGV travels from the dropoff point to Machine 1, from Machine 1 to Machine 2, and from Machine 2 to the dropoff point in time equaling $5\Delta(0.05, n)$, $4\Delta(0.05, n)$, and $6\Delta(0.05, n)$, respectively. The maximum capacity of the AGV is 2, and the buffer capacities of each machine are 4. We provide some sample transition probabilities for the Markov chain associated with Machine 1

$$\begin{aligned}
\text{Case 1:}\quad & \mathsf{P}[\langle 2,1,0,3\rangle | \langle 2,1,0,2\rangle] = p(0,1).\\
\text{Case 3:}\quad & \mathsf{P}[\langle 4,1,0,3\rangle | \langle 4,1,0,2\rangle] = 1.\\
\text{Case 5:}\quad & \mathsf{P}[\langle 2,0,1,0\rangle | \langle 2,1,0,4\rangle] = 1 - p(0,1).\\
\text{Case 10:}\quad & \mathsf{P}[\langle 2,2,0,0\rangle | \langle 2,1,1,8\rangle] = p(0,1).
\end{aligned}$$

The capacity of the AGV when it arrives at machine 1 is Z, and the capacity distribution of the AGV when it arrives at Machine 2 is calculated using Theorem 4.2

$$\mathsf{P}(\beta = 0) = 0.999609, \quad \mathsf{P}(\beta = 1) = 0.000360$$

and

$$\mathsf{P}(\beta = 2) = 0.000031.$$

Then, the transition probabilities for the second Markov chain can be calculated from the distribution of β. The limiting probabilities can then be used to calculate the values of the performance measures, such as the average inventory and the downside risk.

TABLE X
PROBABILITY THAT AGV LEAVES TWO OR MORE JOBS BEHIND FOR GAMMA-DISTRIBUTED INTERARRIVAL TIME

	$\mathsf{P}[D(2) \geq 2]$		
Sys.	Markov Chain	Simulation Model	Error %
1	1	1	0
2	1	1	0
3	1	1	0
4	1	1	0
5	1	1	0
6	1	1	0
7	1	1	0
8	1	1	0
9	1	1	0
10	1	1	0
11	1	1	0
12	1	1	0
13	1	1	0
14	1	1	0
15	1	1	0
16	1	1	0
17	1	1	0
18	1	1	0
19	1	1	0

TABLE XI
SYSTEM (SYS.) PARAMETERS. VALUE OF ψ IS 0.05 FOR EACH SYSTEM, INTERARRIVAL TIMES ARE i. i. d., EXPONENTIALLY DISTRIBUTED AND EACH SYSTEM HAS FIVE MACHINES

	Sys. 20	Sys. 21	Sys. 22	Sys. 23	Sys. 24	Sys. 30
λ_i	1.5	1.9	1.2	1.2	1.5	1.5
$\Delta(\psi, n)$	0.2369	0.187	0.2961	0.2961	0.2369	0.2369
Z	9	10	7	6	8	9
$\bar{X}_i$	3	5	4	4	5	4
$T(0,1)$	4	3	3	4	3	4
$T(1,0)$	16	15	11	14	11	20
$T(0,2)$	8	6	5	6	5	8
$T(2,0)$	12	12	9	12	9	16
$T(3,0)$	10	9	7	8	7	12
$T(0,3)$	10	9	7	10	7	12
$T(4,0)$	12	12	10	12	10	16
$T(0,4)$	8	6	4	6	4	8
$T(5,0)$	16	15	12	16	12	20
$T(0,5)$	4	3	2	2	2	4

TABLE XII
SYSTEM (SYS.) PARAMETERS. VALUE OF ψ IS 0.05 FOR EACH SYSTEM, INTERARRIVAL TIMES ARE i. i. d., GAMMA DISTRIBUTED AND EACH SYSTEM HAS FIVE MACHINES. G DENOTES GAMMA

	Sys. 25	Sys. 26	Sys. 27	Sys. 28	Sys. 29	Sys. 31
$G(n, \lambda)$	(2,2.5)	(2,3.33)	(2,2.5)	(2,2.5)	(3,3.33)	(2,2.5)
$\Delta(\psi, n)$	0.546	0.409	0.546	0.546	0.783	0.546
Z	8	7	7	11	9	7
$\bar{X}_i$	4	3	4	5	4	3
$T(0,1)$	4	4	4	3	2	4
$T(1,0)$	16	16	14	11	10	20
$T(0,2)$	8	8	6	5	4	8
$T(2,0)$	12	12	12	9	8	16
$T(3,0)$	10	10	8	7	6	12
$T(0,3)$	10	10	10	7	6	12
$T(4,0)$	12	12	12	10	8	16
$T(0,4)$	8	8	6	4	4	8
$T(5,0)$	16	16	16	12	10	20
$T(0,5)$	4	4	2	2	2	4

TABLE XIII
AVERAGE NUMBER OF JOBS WAITING AT EACH MACHINE FOR POISSON ARRIVALS

		Sys. 20	Sys. 21	Sys. 22
$\mathsf{E}[W(1)]$	M. C.	2.095019	2.574306	1.963579
	S. M.	2.224527	2.879979	2.205286
	Error %	5.821821	10.61372	10.96034
$\mathsf{E}[W(2)]$	M. C.	2.095019	2.574306	2.456247
	S. M.	2.228725	2.879948	2.607527
	Error %	5.999214	10.61276	5.801665
$\mathsf{E}[W(3)]$	M. C.	2.095019	4.776285	3.905922
	S. M.	2.229172	4.78942	3.903475
	Error %	6.018064	0.274250	0.062687
$\mathsf{E}[W(4)]$	M. C.	2.986211	4.997921	3.998362
	S. M.	2.977772	4.998718	3.999370
	Error %	0.283399	0.015944	0.025200
$\mathsf{E}[W(5)]$	M. C.	2.999972	4.999991	3.999926
	S. M.	2.993839	4.999914	3.999926
	Error %	0.204854	0.001540	0.001400

TABLE XIV
TABLE XIII (*CONTINUED*) AVERAGE NUMBER OF JOBS WAITING AT EACH MACHINE FOR POISSON ARRIVALS

		Sys. 23	Sys. 24
$\mathsf{E}[W(1)]$	M. C.	2.350398	2.235886
	S. M.	2.546467	2.428237
	Error %	7.699648	7.921426
$\mathsf{E}[W(2)]$	M. C.	3.415056	2.977518
	S. M.	3.474673	3.189282
	Error %	1.715758	6.639864
$\mathsf{E}[W(3)]$	M. C.	3.997011	4.825907
	S. M.	3.99684	4.880355
	Error %	0.00427	1.115656
$\mathsf{E}[W(4)]$	M. C.	3.999994	4.99575
	S. M.	3.999902	4.999411
	Error %	0.002300	0.073228
$\mathsf{E}[W(5)]$	M. C.	4	4.999944
	S. M.	3.999926	4.999885
	Error %	0.001850	0.001180

TABLE XV
AVERAGE NUMBER OF JOBS WAITING AT EACH MACHINE FOR GAMMA DISTRIBUTED INTERARRIVAL TIMES

		Sys. 25	Sys. 26	Sys. 27
$\mathsf{E}[W(1)]$	M. C.	3.353113	2.450596	2.618556
	S. M.	3.658632	2.654139	2.639601
	Error %	8.350634	7.668889	0.797279
$\mathsf{E}[W(2)]$	M. C.	3.353113	2.250596	3.145716
	S. M.	3.659242	2.453322	3.175647
	Error %	8.365912	8.2633262	0.942516
$\mathsf{E}[W(3)]$	M. C.	3.993725	2.872796	3.997933
	S. M.	3.999371	2.957465	3.995789
	Error %	0.141172	2.862891	0.053656
$\mathsf{E}[W(4)]$	M. C.	3.999999	2.999990	3.999999
	S. M.	3.999411	2.999966	3.997832
	Error %	0.0147021	0.000800	0.054204
$\mathsf{E}[W(5)]$	M. C.	4	3	4
	S. M.	3.99935	2.999961	3.998242
	Error %	0.016252	0.001300	0.043969

3) Five-Machine Systems: Tables XIII and XIV list the values of $\mathsf{E}[W(n)]$ for $n = 1, 2, \ldots, 5$, and Tables XVII and XVIII list the values of $\mathsf{P}[D(n) \geq 2]$ for $n = 1, 2, \ldots, 5$ for Poisson arrivals. The corresponding values for a gamma-distributed interarrival time are shown in Tables XV and XVI (for the average number waiting) and Tables XIX and XX (for the probability). Tables XXI and XXII present the results from optimization performed with the Markov chain model. Fig. 4

TABLE XVI
TABLE XV (CONTINUED) AVERAGE NUMBER OF JOBS WAITING AT EACH MACHINE FOR GAMMA DISTRIBUTED INTERARRIVAL

		Sys. 28	Sys. 29
	M. C.	2.443493	2.109297
$\mathsf{E}[W(1)]$	S. M.	2.451029	2.115937
	Error %	0.307462	0.313809
	M. C.	2.443493	2.109297
$\mathsf{E}[W(2)]$	S. M.	2.436306	2.122196
	Error %	0.294995	0.607814
	M. C.	4.391445	3.622242
$\mathsf{E}[W(3)]$	S. M.	4.116557	3.50886
	Error %	6.677619	3.231305
	M. C.	4.988988	3.996303
$\mathsf{E}[W(4)]$	S. M.	4.986768	3.995075
	Error %	0.044517	0.030737
	M. C.	4.999924	3.999985
$\mathsf{E}[W(5)]$	S. M.	4.996503	3.996145
	Error %	0.068467	0.096092

TABLE XVII
PROBABILITY THAT AGV LEAVES TWO OR MORE JOBS BEHIND FOR POISSON ARRIVALS

		Sys. 20	Sys. 21	Sys. 22
	M. C.	0	0	0
$\mathsf{P}[D(1) \geq 2]$	S. M.	0	0	0
	Error %	0	0	0
	M. C.	0	0	0
$\mathsf{P}[D(2) \geq 2]$	S. M.	0	0	0
	Error %	0	0	0
	M. C.	0	0.965727	0.973985
$\mathsf{P}[D(3) \geq 2]$	S. M.	0	0.954374	0.969587
	Error %	0	1.189575	0.453595
	M. C.	0.988493	0.999782	0.999961
$\mathsf{P}[D(4) \geq 2]$	S. M.	0.974105	0.999943	0.999931
	Error %	1.477048	0.016100	0.003000
	M. C.	0.99998	0.999999	0.999996
$\mathsf{P}[D(5) \geq 2]$	S. M.	0.998947	0.999997	0.999992
	Error %	0.103408	0.000200	0.000400

TABLE XVIII
TABLE XVII (CONTINUED) PROBABILITY THAT AGV LEAVES TWO OR MORE JOBS BEHIND FOR POISSON ARRIVALS

		Sys. 23	Sys. 24
	M. C.	0	0
$\mathsf{P}[D(1) \geq 2]$	S. M.	0	0
	Error %	0	0
	M. C.	0.886295	0.410045
$\mathsf{P}[D(2) \geq 2]$	S. M.	0.849246	0.399363
	Error %	4.362575	2.674759
	M. C.	0.999475	0.972083
$\mathsf{P}[D(3) \geq 2]$	S. M.	0.999812	0.983722
	Error %	0.033706	1.183159
	M. C.	0.999999	0.999502
$\mathsf{P}[D(4) \geq 2]$	S. M.	0.99999	0.99997
	Error %	0.000900	0.046801
	M. C.	1	0.999995
$\mathsf{P}[D(5) \geq 2]$	S. M.	1	0.999987
	Error %	0	0.000800

shows the cost values for the different capacities and the need for optimization.

Figs. 5 and 6 show the effect of a bigger value of Δ on the value of $\mathsf{E}[W(n)]$ for Systems 30 and 31, respectively. Figs. 7 and 8 show the effect of the same on $P[D(n) > 1]$ for Systems 30 and 31, respectively. As seen from these graphs, the accuracy of the Markov model decreases as Δ increases. Clearly, as Δ decreases, the discrete-time approximation gets closer to its continuous limit [see (2) and (4)].

TABLE XIX
PROBABILITY THAT AGV LEAVES TWO OR MORE JOBS BEHIND FOR GAMMA DISTRIBUTED INTERARRIVAL TIMES

		Sys. 25	Sys. 26
	M. C.	0	0
$\mathsf{P}[D(1) \geq 2]$	S. M.	0	0
	Error %	0	0
	M. C.	0	0
$\mathsf{P}[D(2) \geq 2]$	S. M.	0	0
	Error %	0	0
	M. C.	0.998863	0.992754
$\mathsf{P}[D(3) \geq 2]$	S. M.	0.999782	0.999951
	Error %	0.091920	0.719735
	M. C.	1	0.999999
$\mathsf{P}[D(4) \geq 2]$	S. M.	1	1
	Error %	0	0.0001
	M. C.	1	1
$\mathsf{P}[D(5) \geq 2]$	S. M.	1	1
	Error %	0	0

TABLE XX
TABLE XX (CONTINUED). PROBABILITY THAT AGV LEAVES TWO OR MORE JOBS BEHIND FOR GAMMA DISTRIBUTED INTERARRIVAL TIMES

		Sys. 27	Sys. 28	Sys. 29
	M. C.	0	0	0
$\mathsf{P}[D(1) \geq 2]$	S. M.	0	0	0
	Error %	0	0	0
	M. C.	0	0	0
$\mathsf{P}[D(2) \geq 2]$	S. M.	0	0	0
	Error %	0	0	0
	M. C.	0.999766	0.781874	0.852917
$\mathsf{P}[D(3) \geq 2]$	S. M.	0.999018	0.749082	0.78703
	Error %	0.074873	4.377485	8.371599
	M. C.	1	0.998709	0.999197
$\mathsf{P}[D(4) \geq 2]$	S. M.	0.99941	0.999083	0.99906
	Error %	0.059034	0.037434	0.013712
	M. C.	1	0.999993	0.999997
$\mathsf{P}[D(5) \geq 2]$	S. M.	1	0.999541	0.99906
	Error %	0	0.045220	0.093788

TABLE XXI
OPTIMIZED CAPACITY (Z^*) AND OPTIMAL COST (C^*) FOR SYSTEMS WITH TWO MACHINES. HERE, $c_1 = \$300, c_2 = \$550$, AND $c_3 = \$10000$

System	Z^*	C^*($)
2	4	14375.812
4	3	13466.491
9	8	14436.165
17	7	14007.607
18	7	13910.444

TABLE XXII
OPTIMIZED CAPACITY (Z^*) AND OPTIMAL COST (C^*) FOR SYSTEMS WITH FIVE MACHINES. HERE, $c_1 = \$300, c_2 = \$550$, AND $c_3 = \$10000$

System	Z^*	C^*($)
21	5	23803.5
22	16	20586
24	20	22127
28	10	23747
29	4	21248.5

The major conclusion from our experiments is that the Markov chain model produces a solution with a good quality and does so in a very reasonable amount of computer time, which is less than 2 minutes on a Pentium processor PC with 2-GHz CPU frequency and 250-MG RAM size. The simulation model in comparison takes about four times as much time.

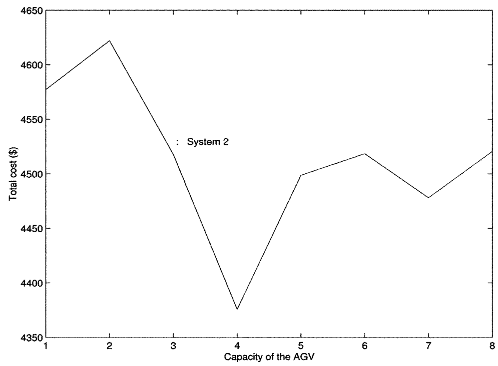

Fig. 4. Total cost versus capacity of AGV for System 2.

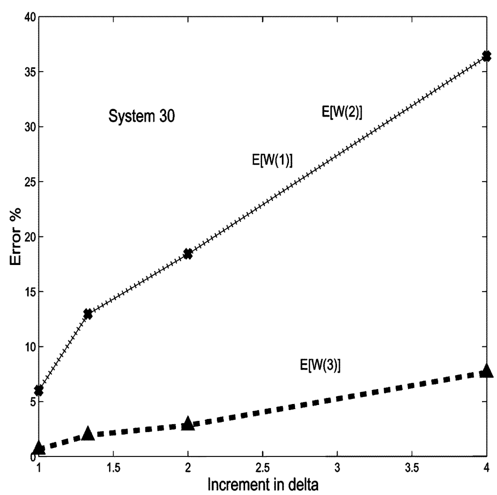

Fig. 5. Effect of a bigger Δ on $\mathsf{E}[W(n)]$ in system 30. Errors in other parameters, i.e., $\mathsf{E}[W(4)]$ and $\mathsf{E}[W(5)]$ are even smaller for plotted values of Δ.

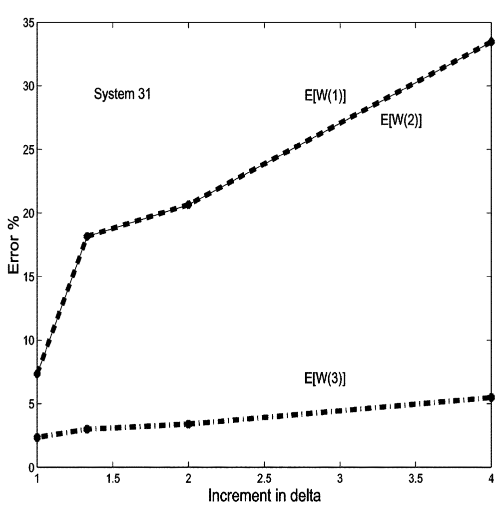

Fig. 6. Effect of a bigger Δ on $\mathsf{E}[W(n)]$ in System 31. Errors in other parameters, i.e., $\mathsf{E}[W(4)]$ and $\mathsf{E}[W(5)]$ are even smaller for plotted values of Δ.

The simulation model and the theoretical model make identical assumptions about the system. The computer codes can be requested from the first author.

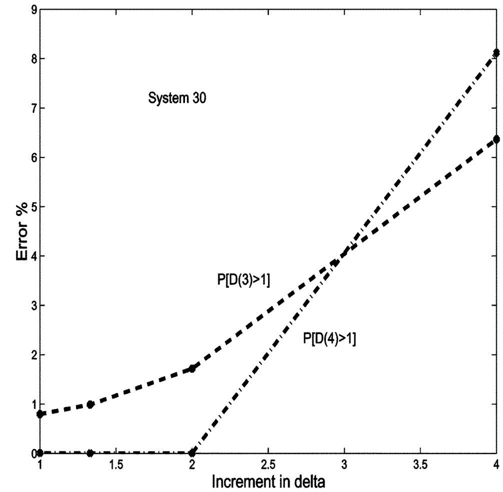

Fig. 7. Effect of a bigger Δ on $P[D(n) > 1]$ in System 30. Errors in other parameters, i.e., $P[D(1) > 1]$, $P[D(2) > 1]$, and $P[D(5) > 1]$ are even smaller for plotted values of Δ.

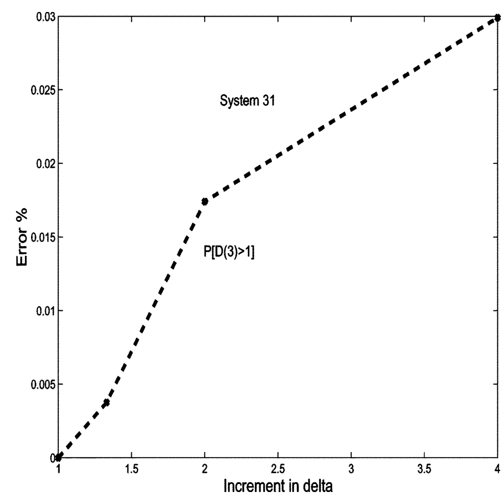

Fig. 8. Effect of a bigger Δ on $P[D(n) > 1]$ in System 31. Errors in other parameters, i.e., $P[D(1) > 1]$, $P[D(2) > 1]$, $P[D(4) > 1]$, and $P[D(5) > 1]$ are even smaller for plotted values of Δ.

VI. Conclusion

In this paper, we studied a version of a problem commonly found in small-scale manufacturing industries, e.g., pharmaceutical firms, which require no more than one AGV that operates in a closed-loop path. We used a Markov chain approach for developing the performance-analysis model. Some special properties of the system were exploited to define a simple structure to the complete problem; the structure allows us to decompose the M-machine system (Markov chain) into M individual systems (Markov chains) and provides us with a mechanism to simplify the computations. This leads to a state-space collapse that is computationally enjoyable. Also, the discretization of the state space in our approach yielded simple expressions for the transition probabilities; a common criticism of many transition-probability models is that they have complicated expressions with multiple integrals (that require numerical integration, which is slow) as transition probabilities.

Some of the advantages of the Markov chain approach are as follows: 1) It is easy to understand; the transition probabilities are characterized by 12 simple conditions. 2) It is simple conceptually because all one needs is $p_\psi(0,n)(=1-F(\Delta(\psi,n)))$ (where $F(\cdot)$ denotes the *cdf* of the job interarrival time), which can be easily calculated for any distribution for the interarrival time of jobs. 3) It produces a state-space collapse thereby making it feasible to compute the limiting probabilities. And finally, 4) it is quite accurate; this is reflected by favorable comparisons with a simulation benchmark.

The main theme of this paper was to develop a simple, although approximate, Markov chain to help study the performance of an AGV system (with one vehicle and a closed-loop path) *and* optimize its capacity. The cost and capacity of the AGV are two parameters that are inextricably linked to each other, and this was an attempt to quantify and analyze this relationship. Extending this model to "dropoff" AGVs, multiple AGVs, or systems that do *not* share some of the properties assumed here should make for exciting topics for further research.

APPENDIX

Theorem A.1: If the interarrival time is triangularly distributed with $T(0,b,c)$, Properties 1 and 2 are satisfied.

Proof: For the triangular distribution, for $\tau < b$

$$\mathsf{P}[W_n(\tau)=0]=\mathsf{P}[S_1>\tau]=1-\frac{\tau^2}{bc}.$$

Also, for $\tau < b$

$$F_1(\tau)\equiv\mathsf{P}[S_1\le\tau]=\frac{\tau^2}{bc}$$

and

$$F_2(\tau)\equiv\mathsf{P}[S_2\le\tau]=\int_0^\tau\frac{(\tau-x)^2}{bc}\frac{2x}{bc}dx=\frac{\tau^4}{6b^2c^2}.$$

Since we are interested in obtaining the limits for τ tending to zero, we do not consider the case for $\tau > b$. Then

$$\mathsf{P}[W_n(\tau)=1]=F_1(\tau)-F_2(\tau)=\frac{\tau^2}{bc}-\frac{\tau^4}{6b^2c^2}.$$

Then, it follows that

$$\lim_{\tau\to0}\frac{\sum_{i=2}^{\infty}\mathsf{P}[W_n(\tau)=i]}{\mathsf{P}[W_n(\tau)=0]}=\lim_{\tau\to0}\frac{\frac{\tau^4}{6b^2c^2}}{1-\frac{\tau^2}{bc}}=0.$$

Property 1 is thus satisfied. Similarly, for property 2, we have

$$\lim_{\tau\to0}\frac{\sum_{i=2}^{\infty}\mathsf{P}[W_n(\tau)=i]}{\mathsf{P}[W_n(\tau)=1]}=\lim_{\tau\to0}\frac{\frac{\tau^4}{6b^2c^2}}{1-\frac{\tau^2}{bc}-\frac{\tau^4}{6b^2c^2}}=\lim_{\tau\to0}\frac{\frac{2\tau^2}{b^2c^2}}{\frac{2}{bc}-\frac{2\tau^2}{b^2c^2}}$$

applying L'Hospital's rule twice $=0$.

■

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