Lecture Abstracts.
Stochastic Fluid Mechanics.

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Lecture 1.
Basic existence and uniqueness results for deterministic and stochastic 2–d and 3–d Navier–Stokes equations.

We first discuss some background knowledge as well as recent progresses in 2–d and 3–d Navier–Stokes equations. We then show basic existence and uniqueness results for deterministic and stochastic 2–d Navier–Stokes equations.

References

[1] Kuksin, S., Shirikyan, A., *Mathematics of 2–dimensional turbulence*. Cambridge Tracts in Mathematics, **194**, November 2012.

[2] Šverák, V., Lecture notes of *Selected Topics in Fluid Mechanics*, 2011–2012.

[3] Šverák, V., Lecture notes of *Topics in PDE*, Spring 2014, personal notes by the author of these lectures.

[4] Flandoli, F., Dissipativity and invariant measures for stochastic Navier–Stokes equations. *NoDEA*, **1**, pp. 403–423, 1994.

[5] Elgindi, T., Hu, W., Šverák, V., Asymptotics for a model in 2–d turbulence. Preprint.

[6] Mikulevicius, R., Rozovskii, B.L., Stochastic Navier–Stokes equations for turbulent flows. *SIAM Journal of Mathematical Analysis*, **35**(5), pp.1250–1310, 2004.

[7] Cafferelli, L., Kohn, R., Nirenberg, J., Partial regularity of suitable weak solutions of the Navier–Stokes equations. *Communications in Pure and Applied Mathematics*, **35** (1982), no. 6, pp. 771–831.

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[8] Jia, H., Šverák, V., Local–in–space estimates near initial time for weak solutions of the Navier–Stokes equations and forward self–similar solutions. *Inventions Mathematica*, Online. `arXiv:1204.0529`

[9] Jia, H., Šverák, V., Are the incompressible 3d Navier–Stokes equations locally ill–posed in the natural energy space? `arXiv:1306.2136`

Lecture 2.

Existence and uniqueness of invariant measures for 2–d hydrodynamical systems subject to degenerate random forcing.

We present a model problem in finite dimensions showing the existence and uniqueness of invariant measure for general white–noise driven stochastic systems with hydrodynamical background. We will also mention the result as well as difficulties for infinite dimensional systems corresponding to general semilinear evolutionary SPDEs. Application of the theoretical method includes hydrodynamical systems such as the 2–d Navier–Stokes system, the 2–d Boussinesq system, and a 2–d Euler system subject to fractional dissipation.

References

[1] Hairer, M., Mattingly, J.C., Ergodicity of the 2-d Navier–Stokes equations with degenerate stochastic forcing. *Annals of Mathematics*, (2), **164**(3), pp. 567–600, 2009.

[2] Hairer, M., Mattingly, J.C., A theory of hypoellipticity and unique ergodicity for semilinear parabolic SPDEs. *Electronic Journal of Probability*, **16** (2011), pp. 658–738.

[3] Kuksin, S., Shirikyan, A., *Mathematics of 2–dimensional turbulence*. Cambridge Tracts in Mathematics, **194**, November 2012.

[4] Constantin, P., Glatt–Holtz, N., Vicol, V., Unique ergodicity for fractionally dissipated, stochasticaly forced 2–d Euler equations. `arXiv:1304.2022[MATH.AP]`

[5] Földes, J., Glatt–Holtz, N., Richards, G., Thomann, E., Ergodic and mixing properties of the Boussinesq equations with a degenerate random forcing. `arXiv:1311.3620v1`

[6] Glatt–Holtz, N., Notes on statistically invariant states in stochastically driven fluid flows. available at

`http://www.math.vt.edu/people/negh/research/index.html`

Lecture 3.
Inviscid limit and related problems in turbulence.

Asymptotic problems of sending the viscosity $\nu \to 0$ will be discussed in this lecture. In particular, we discuss the application of de Giorgi–Nash–Moser iteration method to stochastic Navier–Stokes equations. We show the corresponding invariant mesure is concentrated in L^∞ in vorticity space. The relationship between inviscid limit and turbulence will be explained.

References

[1] Glatt–Holtz, N., Šverák, V., Vicol, V., On inviscid limits for the stochastic Navier-Stokes equation. `arXiv:1302.0542[MATH.AP]`

[2] Kuksin, S., Shirikyan, A., *Mathematics of 2–dimensional turbulence.* Cambridge Tracts in Mathematics, **194**, November 2012.

[3] Arnold, V.I., Meshchalkin, L.D., Kolmogorov seminar on selected questions of analysis 1958–1959, *Russian Mathematical Surverys*, **15**, 1, pp. 247–250. (1960)

Lecture 4.
The 2–d deterministic and stochastic Euler equations and related problems.

The L^∞–well posedness (in vorticity formulation) of 2–d deterministic Euler equations will be discussed. Recent results in singularity formation in the sense of double exponential growth of the gradient of the vorticity at the boundary will be mentioned. The area–preserving scheme of adding the stochastic noise to the equation will be considered. Well–posedness results in the case of stochastic equation will also be discussed.

References

[1] Bertozzi, A., Majda, A., Vorticity and incompressible flow. *Cambridge Texts in Applied Mathematics*, **27**, Cambridge University, 2002.

[2] Kiselev, A., Šverák, V., Small scale creation for solutions of the incompressible 2–dimensional Euler equation. *Annals of Mathematics* **180** (2014), pp. 1205–1220.

[3] Brzeźniak, Z., Flandoli, F., Maurelli, M., Existence and uniqueness for stochastic 2D Euler flows with bounded vorticity. `arXiv:1401.5938`

Lecture 5.
Motion of incompressible ideal fluids from group theoretic and Hamiltonian dynamical point of view.

Classical mechanics from group theoretic point of view will be discussed, including the motion of incompressible ideal fluids. The Hamiltonian formulation of Euler equations will be presented. If time permits, we will also discuss finite dimensional model problems and some open questions related to 2–d turbulence.

References

[1] Arnold, V.I., *Mathematical methods of classical mechanics*, Springer, 1978.

[2] Arnold, V.I., Khesin, B., *Topological methods in hydrodynamics*, Springer, 1998.

[3] Hu, W., Šverák, V., Random motion along co–adjoint orbits. Preprint.

[4] Šverák, V., Lecture notes of *Selected Topics in Fluid Mechanics*, 2011–2012.