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Preface

In this monograph we present recent work of last four years of the author in discrete and fractional analysis. It is the natural outgrowth of his related publications. Chapters are self-contained and can be read independently and advanced courses can be taught out of this book. An extensive list of references is given per chapter.

The topics covered are diverse. We introduce the right delta and right nabla fractional calculus on time scales. We continue with right delta and right nabla discrete fractional calculus in the Caputo sense. Then we give representations formulae of functions on time scales and we present Ostrowski type inequalities, Landau type inequalities, Grüss type and comparison of means inequalities, all these over the time scales. We continue with integral operator inequalities and their multivariate vectorial versions using convexity of functions again all these over time scales. It follows Grüss and Ostrowski type inequalities involving s-convexity of functions, we examine also the general case when we involve several functions. Then we present general fractional Hermite–Hadamard type inequalities using m-convexity and (s, m)-convexity. We finish monograph by introducing the reduction method in fractional calculus and study its connection to fractional Ostrowski type inequalities.

This book's results are expected to find applications in many areas of pure and applied mathematics, especially in difference equations and fractional differential equations. As such this monograph is suitable for researchers, graduate students, and seminars of the above subjects, also to be in all science libraries.

The preparation of book took place during 2014–2015 in Memphis, Tennessee, USA.

I would like to thank Professor Razvan Mezei, of Lenoir-Rhyne University, for checking and reading the manuscript.

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April 1, 2015

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