

I would like to announce you the publication of the monograph  
**"Nonlinear second order evolution equations of monotone type  
and applications"** at Pushpa Publishing House, 2007;

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You can read below the preface and the contents of the book.

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## PREFACE

The book is devoted to some second order differential equations of the form  $pu'' + ru' \in Au + f$  on  $(0, T)$  or on  $[0, \infty)$ . The equation is governed by a maximal monotone operator  $A$  in a real Hilbert space  $H$ . Some classes of elliptic partial differential equations can be particularly written under this form. Different boundary conditions of monotone type are associated.

This work presents various results related to the above equations, their discrete variants and some applications to optimization, internal schemes of approximation and singular perturbation problems. Particular cases of some partial differential equations are discussed.

In the study of these problems, one combines the methods of Nonlinear Analysis with the theory of nonlinear semigroups of contractions in Hilbert spaces. The general theory has applications to singular perturbation problems, to optimization and to Numerical Analysis.

The existence of the solution to the above equation with different boundary conditions is proved. Its uniqueness up to an additive constant is also established. The square root of the operator  $A$  is defined and a semigroup of contractions is constructed. In the case of the subdifferential mappings, one proves the equivalence with some minimization problems. The asymptotic behavior of the solution of the equations on half-axis is analyzed. Further

on, we obtain the continuous dependence on data of the solutions and give some applications to optimization and to internal approximation schemes.

Next, we study the discrete variants of the above evolution equations. One finds some classes of second order difference inclusions of monotone type. The existence theory is developed here too and the asymptotic behavior of the solution is studied. If the operator  $A$  is a subdifferential mapping, then the problem is equivalent with an optimization problem. Applications to internal approximation schemes are given. Second order differential equations with one initial condition arises in monetary models.

Finally we present some applications to singularly perturbed problems. The solution  $v$  of the semilinear heat equation is compared with the solution  $v_\varepsilon$  of an elliptic regularization. Under some specific hypotheses, we construct an asymptotic approximation for  $v_\varepsilon$  and find the order of accuracy of the difference  $v_\varepsilon - v$ . Thus, the solution  $v$  of the heat equation is approximated by the solution  $v_\varepsilon$  of its elliptic regularization, which is more regular. This is a strong motivation for the study of the above mentioned second order evolution equations associated with monotone operators. Similarly, an asymptotic approximation is studied for the linear heat equation with nonlinear boundary conditions. The last application involves the semilinear telegraph system. The singular perturbation problem studied here is of elliptic-hyperbolic type.

The subject treated in this book was investigated by many mathematicians, as V. Barbu, H. Brézis, R. E. Bruck, E. Mitidieri, G. Moroşanu, N. Pavel, S. Reich, I. Shafrir, L. Véron, X. Xue, etc. Some contributions are due to the author.

The book can be read by researchers, scientists and PhD students who study nonlinear evolution equations, Nonlinear Analysis, Partial Differential Equations, difference equations, perturbation theory, asymptotic analysis and their applications.

Part of the content of this book was presented in the work "Ecuatii Diferenţiale Neliniare de Ordinul al Doilea în Spaţii Hilbert" (in Romanian), by N. Apreutesei and G. Moroşanu.

Special thanks to Professors V. Barbu and G. Moroşanu for their contribution in my professional activity.

Iasi, June 2006  
N. C. Apreutesei

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