Chapter 1, Section 4

2. \[ \text{Area} = \text{length} \times \text{width} \]
   \[ = w \times 2w \]
   \[ A = 2w^2 \]

6. \[ \text{Suppose length of fence} = F. \]
   \[ F = 2l + 2w. \]
   \[ \text{We know Area} = 3600 \text{m}^2 = lw, \text{ so} \]
   \[ l = \frac{3600}{w}. \text{ Substitute to get} \]
   \[ F = \frac{7200}{w} + 2w \]

<table>
<thead>
<tr>
<th>( w )</th>
<th>( F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>400</td>
</tr>
<tr>
<td>40</td>
<td>260</td>
</tr>
<tr>
<td>60</td>
<td>240</td>
</tr>
<tr>
<td>80</td>
<td>250</td>
</tr>
<tr>
<td>100</td>
<td>272</td>
</tr>
</tbody>
</table>

Looks like \( F \) is smallest if \( w = 60 \). Then \( l = \frac{3600}{60} = 60 \).

The dimensions of approximately \( 60 \text{m} \times 60 \text{m} \) should require the smallest amount of fencing.

8. \[ \text{Volume} = 1500 \text{ in}^3 = b^2 \cdot h. \text{ Find surface area} \]
   \[ \text{in terms of} \ b. \]
   \[ \text{SA} = b^2 + b^2 + bh + bh + bh + bh \]
   \[ \text{SA} = 2b^2 + 4bh \]
   \[ \text{Need to eliminate} \ h. \ h = \frac{1500}{b^2}. \text{ Substitute.} \]
   \[ \text{SA} = 2b^2 + 4b(\frac{1500}{b^2}) \]
   \[ \text{SA} = 2b^2 + \frac{6000}{b} \]
Chapter 1, Section 4

Cost for side = 0.02 cents/in^2.
Cost for top & bottom = 0.04 cents/in^2.
Volume = $4\pi r^2 h$.

If we cut open the can, we can lay it flat like this:

Cost = 0.02 (area of rectangle) + 0.04(2)(area of one circle).

We need the length of the rectangle, which is actually the circumference of the circle.

\[ l = 2\pi r. \]

Cost = 0.02(2\pi r)(h) + 0.08(\pi r^2).

We want to eliminate h so cost is only in terms of r.

We know Volume = $4\pi = \pi r^2 h$, so \( h = \frac{4}{r^2} \).

Cost = 0.04 \pi r (\frac{4}{r^2}) + 0.08 \pi r^2

Cost = 0.16 \pi / r + 0.08 \pi r^2

15. Population grows at a rate proportional to size of population.
rate of growth \( r = kp \) where \( k \) is constant and \( p \) is the population size.

17. \( t = \) temp of object. \( r = \) rate temp changes. \( a = \) temperature of surrounding medium (this is called the ambient temp).
\( r = k (t-a) \)
(This particular relationship is called "Newton's Law of Cooling").
Chapter 1, Section 4

35. \( S(p) = 4p + 200 \), \( D(p) = -3p + 480 \). Equilibrium is where \( S(p) = D(p) \), so solve \( 4p + 200 = -3p + 480 \).

\[ 7p = 280 \]
\[ p = 40. \]

When \( p = 40 \), \( S(40) = 360 \) units and \( D(40) = 360 \).

(Not a coincidence! The idea of equilibrium means \( S = D \).)

38. At a speed of 72 kph, after 40 minutes, our hero has a lead of

\[ 40 \text{ min} \times \frac{1 \text{ hour}}{60 \text{ min}} \times \frac{72 \text{ km}}{1 \text{ hour}} = 48 \text{ kilometers}. \]

Total distance is 83.8 km, so he still must travel 35.8 km.

This will take him \( 35.8 \text{ km} \times \frac{1 \text{ hour}}{72 \text{ km}} = 0.4972 \) hours.

The pursuers have to travel 83.8 km at 168 kph, which will take them \( 83.8 \text{ km} \times \frac{1 \text{ hour}}{168 \text{ km}} \approx 0.4988 \) hours.

When our hero reaches the border, the bad guys are just barely behind him, so he gets away.

44. Let \( x \) = # checks written per month. Cost at bank 1 is \( C_1 = 12 + 0.10x \), and at bank 2 is \( C_2 = 10 + 0.14x \).

1st bank is better if \( 12 + 10x < 10 + 14x \)

\[ 2 < 0.04x \]
\[ 50 < x. \]

1st bank is better if you write more than 50 checks per month.

If you write less than 50 and bank 2 is better.
Chapter 1, Section 5

1. \(\lim_{x \to a} f(x) = b\)  
2. \(\lim_{x \to a} f(x) = b\)  
3. \(\lim_{x \to a} f(x) = b\)  
4. \(\lim_{x \to a} f(x) \) does not exist  
5. \(\lim_{x \to a} f(x) \) does not exist  
6. \(\lim_{x \to a} f(x) = b\)

10. \(\lim_{x \to 0} (1-5x^3) = \lim_{x \to 0} 1 - 5(\lim_{x \to 0} x)^3\) since it's a polynomial.  

\[= 1 - 5(0) = 1.\]

\[\lim_{x \to 1} \frac{2x+3}{x+1} = \frac{2+3}{1+1} = \frac{5}{2}\]

18. \(\lim_{x \to 3} \frac{9-x^2}{x-3} = \lim_{x \to 3} \frac{(3-x)(3+x)}{x-3} = \lim_{x \to 3} -(3+x) = -6\)

24. \(\lim_{x \to 1} \frac{x^2+4x-5}{x^2-1} = \lim_{x \to 1} \frac{(x+5)(x-1)}{(x+1)(x-1)} = \lim_{x \to 1} \frac{x+5}{x+1} = \frac{6}{2} = 3\)

26. \(\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} = \lim_{x \to 9} \frac{(\sqrt{x} - 3)(\sqrt{x} + 3)}{(x - 9)(\sqrt{x} + 3)} = \lim_{x \to 9} \frac{x-9}{(x-9)(\sqrt{x}+3)}\)

\[= \lim_{x \to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{6}\]

28. \(\lim_{x \to 5^-} \frac{\sqrt{2x-1} - 3}{x-5} = \lim_{x \to 5^-} \frac{(\sqrt{2x-1} - 3)(\sqrt{2x-1} + 3)}{(x-5)(\sqrt{2x-1} + 3)}\)

\[= \lim_{x \to 5^-} \frac{2x-1-9}{(x-5)(\sqrt{2x-1}+3)} = \lim_{x \to 5^-} \frac{2(x-5)}{(x-5)(\sqrt{2x-1}+3)}\]

\[= \lim_{x \to 5^-} \frac{2}{\sqrt{2x-1}+3} = \frac{2}{\sqrt{9}+3} = \frac{1}{3}\]
Chapter 1, Section 5

30. \( f(x) = \begin{cases} \frac{1}{x-1} & x < -1 \\ x^2 + 2x & x \geq -1 \end{cases} \)

\[ \lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} \frac{1}{x-1} = -\frac{1}{2} \]

\[ \lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (x^2 + 2x) = 1 - 2 = -1 \]

31. \[
\begin{array}{c|cccc}
\hline
x & 1 & 0.1 & 0.01 & 0.001 & 0.0001 \\
\hline
1000(1 + 0.09x)^{\frac{1}{2}} & 1090 & 1093.73 & 1094.13 & 1094.17 & 1094.17 \\
\hline
\end{array}
\]

37. \( \lim_{x \to 0} f(x) \) does not exist, keeps bouncing between 1 and -1.

38. \( \lim_{x \to 0} g(x) = 0 \) since graph gets shorter and shorter as \( x \to 0 \).
4. \( f(x) = \frac{2x-4}{3x-2} \). This is continuous at \( x = 2 \): 
\( f(2) = \frac{0}{4} = 0 \), 
\( \lim_{{x \to 2}} f(x) = 0 \). (The discontinuity is at \( x = \frac{2}{3} \)).

6. \( f(x) = \frac{2x+1}{3x-6} \). This is discontinuous at \( x = 2 \) since 
\( f(2) = \frac{5}{6} \) and is not defined.

10. \( f(x) = \begin{cases} 
  x+1 & x < 0 \\
  x-1 & x \geq 0
\end{cases} \). 
\( f(0) = 0 - 1 = -1 \). 
\( \lim_{{x \to 0^+}} f(x) = 0 - 1 = -1 \) \( \lim_{{x \to 0^-}} f(x) = 0 + 1 = 1 \) \text{ not continuous at } x = 0.

11. \( f(x) = \begin{cases} 
  x^2 + 1 & x \leq 3 \\
  2x + 4 & x > 3
\end{cases} \). 
\( f(3) = 3^2 + 1 = 10 \) 
\( \lim_{{x \to 3^+}} f(x) = 2(3) + 4 = 10 \) \( \lim_{{x \to 3^-}} f(x) = 3^2 + 1 = 10 \) \text{ continuous at } x = 3.

14. \( f(x) = x^5 - x^3 \) is continuous everywhere.

18. \( f(x) = \frac{x^2 - 1}{x+1} \) is discontinuous at \( x = -1 \)

22. \( f(x) = \frac{x^2 - 2x + 1}{x^2 - x - 2} = \frac{(x-1)(x-1)}{(x-2)(x+1)} \) discontinuous at \( x = 2, -1 \).

24. \( f(x) = \begin{cases} 
  x^2 & x \leq 2 \\
  9 & x > 2
\end{cases} \) Discontinuous at \( x = 2 \) : 
\( \lim_{{x \to 2^-}} f(x) = 2^2 = 4 \) 
\( \lim_{{x \to 2^+}} f(x) = 9 \). 
Graph "breaks" at \( x = 2 \).
32. \( f(x) = x(1 + \frac{1}{x}) \) is continuous on \( 0 < x < 1 \), but \( f(x) \) does not exist at \( x = 0 \). So on \( 0 \leq x \leq 1 \), \( f(x) \) is discontinuous at \( x = 0 \).

35. Show that \( 3\sqrt{x} = x^2 + 2x + 1 \) must have a solution on \( 0 \leq x \leq 1 \). Define \( f(x) = x^2 + 2x + 1 - 3\sqrt{x} \). For original equation to have a solution, we need \( f(x) = 0 \).

\[ f(0) = -1 \quad \text{and} \quad f(1) = 1 \]

Since \( -1 < 0 < 1 \), and \( f(0) = -1, \ f(1) = 1 \), using the intermediate value property there is some number \( c \) so that \( 0 < c < 1 \) and \( f(c) = 0 \). (Since \( f(x) \) is continuous).

38. The minute hand of a clock moves in a continuous fashion. Since the hour hand moves much slower, there is a time when the minute hand is behind the hour hand, as well as a time when it's ahead. Since the motion is continuous, there must also be a time when the hands coincide.