Section 7.1 - Functions of Several Variables

2. \[ f(x,y) = \frac{3x + 2y}{2x + 3y}, \quad f(1,2) = \frac{3 + 4}{2 + 6} = \frac{7}{8} \]
   \[ f(-4,6) = \frac{-12 + 12}{-8 + 18} = \frac{0}{10} = 0 \]

6. \[ f(x,y) = xy e^{xy} \]
   \[ f(1,\ln 2) = (1)(\ln 2) e^{(1)(\ln 2)} = (\ln 2)(2) = 2\ln 2 \]
   \[ f(\ln 3, \ln 4) = (\ln 3)(\ln 4) e^{\ln 3 \cdot \ln 4} = (\ln 3)(\ln 4)(3 \ln 4) \]

10. \[ g(x,y,z) = (x+y) e^{yz} \]
    \[ g(1,0,-1) = (1+0) e^{(0)(-1)} = e^0 = 1 \]
    \[ g(1,1,2) = (1+1) e^{(1)(2)} = 2e^2 \]

14. \[ f(x,y) = \sqrt{9-x^2-y^2} \]
    Domain is all \((x,y)\) so that \(9-x^2-y^2 \geq 0\).
    \[ 9 \geq x^2 + y^2, \quad x^2 + y^2 \leq 9. \text{ This is the inside of a circle of radius 3, centered at the origin, including the border.} \]

18. \[ f(x,y) = \frac{e^{xy}}{\sqrt{x-2y}} \]
    Domain is all \((x,y)\) so that \(x-2y > 0\).
    (can't = 0 or denom will be zero).
    \[ x > 2y, \quad y = \frac{1}{2}x \]
    \[ y < \frac{1}{2}x \]
    This is the half of the xy-plane below the line \( y = \frac{1}{2} x \), not including the line itself.
Section 7.1 - (cont)

20. \( f(x, y) = x^2 + y \)
   
   \( c = 0 \)
   \( 0 = x^2 + y \)
   \( y = -x^2 \)
   
   \( c = 4 \)
   \( 4 = x^2 + y \)
   \( y = 4 - x^2 \) \( \{ \text{parabolas} \} \)
   
   \( c = 9 \)
   \( 9 = x^2 + y \)
   \( y = 9 - x^2 \)

24. \( f(x, y) = ye^x \)
   
   \( c = 0 \)
   \( 0 = ye^x \) \( \leftarrow \text{we know } e^x \neq 0, \text{ so } y = 0 \) \( \text{(the x-axis)} \)
   
   \( c = 1 \)
   \( 1 = ye^x \)
   \( y = e^{-x} \)

28. \( b = \# \text{ business calculators, } g = \# \text{ graphing calculators. (\# per month)} \)

   a) \( \text{production cost } = C(b, g) = 40g + 20b \)

   b) \( C(800, 500) = 40(500) + 20(800) = 20000 + 16000 = \$36000. \)

   c) \( g \) is now 550 instead. Find \( b \) so that \( C = 32000 \)
      \( 36000 = 40(550) + 20b = 22000 + 20b \)
      \( 14000 = 20b \), so \( b = 700. \) Reduce the number of business
      calculators by 100 (from 800 to 700) to keep cost same.
Section 7.1-(cont)

32. \( x = \# \text{ machines, } y = \# \text{ worker hours}. \ Q(x,y) = 10 \ xy \text{ cell phones.} \)

\[ 1000 = 10 \ xy \]

\[ xy = 100, \ y = \frac{100}{x} \text{ is the relationship resulting in 1000 cell phones.} \]

34. \( \text{price} = \$ A \text{ per unit.} \ x = \text{thousand dollars on development,} \ y = \text{thousand dollars spent on promotion.} \ # \text{units bought} = \frac{320y}{y+2} + \frac{160x}{x+4}. \)

\( \text{Costs are} \ $50 \text{ per unit. Find profit.} \)

\[
\text{Profit} = P(x,y) = \text{revenue} - \text{cost} = A \left( \frac{320y}{y+2} + \frac{160x}{x+4} \right) - \left[ (50) \left( \frac{320y}{y+2} + \frac{160x}{x+4} \right) + x+y \right] = (A-50) \left( \frac{320y}{y+2} + \frac{160x}{x+4} \right) - x-y.
\]

44. Ice is thickest where level curve over land has the largest \( c \)-value. Looks like it's in Antarctica, over in the bottom left side of the picture, about 4000 meters thick.

The warmest sea next to land with ice on it is harder to see. It's 12°C near the coast of ice-bound Alaska. Can't tell if Eastern Australia is ice-bound from the map, but the ocean is 18°C or 19°C there ... (etc.)