NAME $\underline{\text{KEY}}$

Math 12
Test 1
Spring 2013

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1. Using the definition of the derivative, find $f'(x)$ if $f(x)=\sqrt{5-3x}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{5-3(x+h)}-\sqrt{5-3x}}{h} \cdot \frac{\sqrt{5-3(x+h)}+\sqrt{5-3x}}{\sqrt{5-3(x+h)}+\sqrt{5-3x}}$$

$$= \lim_{h\to 0} \frac{5-3(x+h)-(5-3x)}{h(\sqrt{5-3(x+h)}+\sqrt{5-3x})} = \lim_{h\to 0} \frac{\cancel{5}-\cancel{3x}-3h-\cancel{5}+\cancel{3x}}{h(\sqrt{5-3(x+h)}+\sqrt{5-3x})}$$

$$= \lim_{h\to 0} \frac{-3\cancel{h}}{\cancel{h}(\sqrt{5-3(x+h)}+\sqrt{5-3x})} = \frac{-3}{2\sqrt{5-3x}}$$

2. Evaluate the following limits. If any of them do not exist, EXPLAIN why not ("because it's undefined" and "denominator is zero" are not sufficient explanations).

(a) $\displaystyle\lim_{x\to 3} \frac{x^2-2x-3}{9-x^2} = \lim_{x\to 3} \frac{(x-3)(x+1)}{(3-x)(3+x)} = \lim_{x\to 3} \frac{-(x+1)}{3+x} = \frac{-4}{6} = \frac{-2}{3}$

fill in, get $\frac{0}{0}$ ...

(b) $\displaystyle\lim_{x\to 3} \frac{x^2+5x+6}{x+3} = \frac{9+15+6}{6} = \frac{30}{6} = 5$

(c) $\displaystyle\lim_{x\to 2} \frac{4}{(x-2)^2} = \infty$

fill in, get $\frac{4}{0}$. Use chart.

x	y
1	$4/1 = 4$
1.5	$4/\frac{1}{4} = 16$
1.9	$4/\frac{1}{100} = 400$
3	$4/1 = 4$
2.5	$4/\frac{1}{4} = 16$
2.1	$4/\frac{1}{100} = 400$

$\lim_{x\to 2^-} f(x) = \infty$

$\lim_{x\to 2^+} f(x) = \infty$

3. A company produces cell phones at a cost of \$50 each. Consumers will purchase x phones per week if the price is set at $p(x) = 200 - x$ dollars.

a) Find a function for weekly Profit.

Profit = Revenue − Cost = price · quantity − cost

Profit = $(200-x)(x) - (50)(x) = 200x - x^2 - 50x$

$P(x) = 150x - x^2$

b) Find a function for weekly Marginal Profit.

$mP(x) = P'(x) = 150 - 2x$

c) Suppose the company currently produces 70 phones each week. Assume the goal of the company is to maximize profit. Based on the marginal profit at this level of production, should production be increased, decreased, or remain the same? Explain your answer in words.

$P'(70) = 150 - 2(70) = 150 - 140 = 10.$

This means that if the company produces the 71st cell phone, profit will go up approximately \$10. Since profit will increase, production should be increased.

4. Find $f'(x)$ (do not simplify!) if :

a) $f(x) = \dfrac{3x^6}{\sqrt{x}} + 4x^{-3} + \dfrac{x^2-1}{2x} + 5x - 2 \quad = \quad 3x^6x^{-1/2} + 4x^{-3} + \frac{1}{2}(x^2-1)(x^{-1}) + 5x - 2$

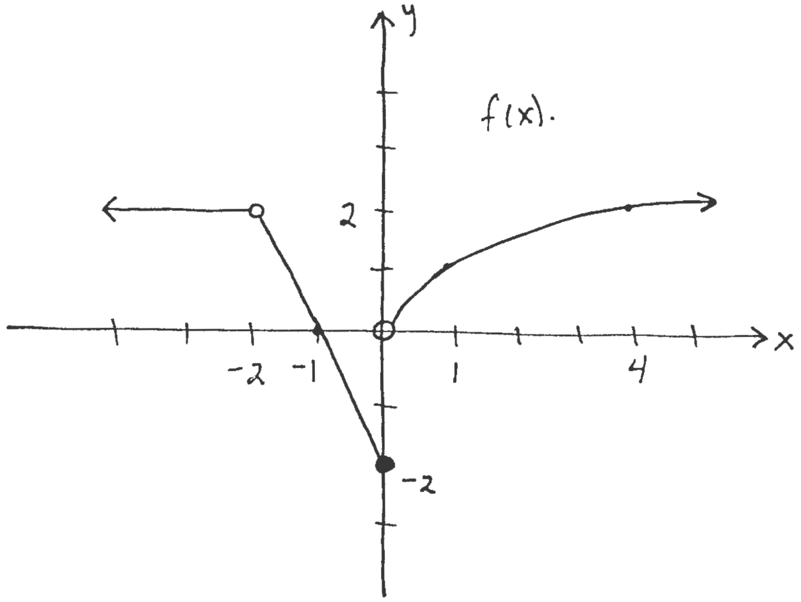
$f(x) = 3x^{11/2} + 4x^{-3} + \frac{1}{2}(x - x^{-1}) + 5x - 2$

$f'(x) = \frac{33}{2}x^{9/2} - 12x^{-4} + \frac{1}{2}(1 + x^{-2}) + 5$

b) $f(x) = \dfrac{\sqrt[3]{x^2} - 5x^3 + 7}{2x-5} \quad = \quad \dfrac{x^{2/3} - 5x^3 + 7}{2x-5}$

$$f'(x) = \frac{\left(\frac{2}{3}x^{-1/3} - 15x^2\right)(2x-5) - (x^{2/3} - 5x^3 + 7)(2)}{(2x-5)^2}$$

5. Sketch a nice big graph of $f(x)=\begin{cases} 2 & x<-2 \\ -2(x+1) & -2<x\le 0 \\ \sqrt{x} & 0<x \end{cases}$. Be sure to clearly label points and axes. Under your graph, list the interval(s) where $f(x)$ is continuous.



f(x) is continuous on $(-\infty,-2) \cup (-2,0) \cup (0,\infty)$

6. Find the equation of the line tangent to $f(x)=\dfrac{(3x-4)(x^3-5)}{4x^2-2x+1}$ at the point where $x=1$.

Point : $x=1$

$y = \dfrac{(-1)(-4)}{4-2+1} = \dfrac{4}{3}$ $(1, 4/3)$

Slope : $f'(x) = \dfrac{[3(x^3-5)+(3x-4)(3x^2)](4x^2-2x+1)-(3x-4)(x^3-5)(8x-2)}{(4x^2-2x+1)^2}$

$m = f'(1) = \dfrac{(3(-4)+(-1)(3))(4-2+1)-(-1)(-4)(6)}{(4-2+1)^2}$

$= \dfrac{(-12-3)(3)-24}{9} = \dfrac{-45-24}{9} = \dfrac{-69}{9} = \dfrac{-23}{3}$

Line : $y - 4/3 = \dfrac{-23}{3}(x-1)$

7. Consider the graph of the function $f(x)$ given below.

(a) Find $\lim_{x\to 0} f(x)$. $= 2$

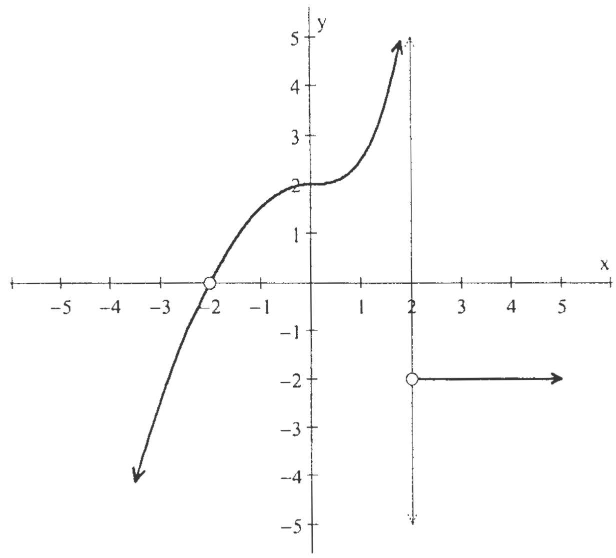
(b) Find $\lim_{x\to 2^-} f(x)$. $= \infty$

(c) Find $\lim_{x\to 2^+} f(x)$. $= -2$

(d) Find $\lim_{x\to 2} f(x)$. DNE

(e) Find $\lim_{x\to -2} f(x)$. $= 0$

(f) Find $\lim_{x\to +\infty} f(x)$. $= -2$



8. Find the equation of the line perpendicular to $3x-5y=10$ which contains the point $(2,-1)$.

$3x-5y=10$

$-5y=-3x+10$

$y=\frac{3}{5}x-2$

old $m = 3/5$

new $m = -5/3$

<u>line</u>: $y+1=\frac{-5}{3}(x-2)$