NAME KEY

Math 12
Test 2
Spring 2013

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Suppose $f(x)=\dfrac{x^2}{x^2-4}$. Find all intervals where $f(x)$ is concave up and where it is concave down (interval notation, please). List the inflection points.

$$f'(x)=\frac{(2x)(x^2-4)-(x^2)(2x)}{(x^2-4)^2}=\frac{2x^3-8x-2x^3}{(x^2-4)^2}=\frac{-8x}{(x^2-4)^2}$$

$$f''(x)=\frac{(-8)(x^2-4)^2+8x(2)(x^2-4)(2x)}{(x^2-4)^4}=\frac{-8x^2+32+32x^2}{(x^2-4)^3}$$

$$=\frac{24x^2+32}{(x^2-4)^3}$$

IN: $x=\pm 2$

f'': $+$ on $(-\infty,-2)$, $-$ on $(-2,2)$, $+$ on $(2,\infty)$

conc up on $(-\infty,-2)\cup(2,\infty)$
conc down on $(-2,2)$
no inflection points since $f(2)$ and $f(-2)$ are undefined

2. For the following functions, find all horizontal and vertical asymptotes (remember that an asymptote is a LINE, not a number). If there are no asymptotes, say so.

a) $f(x)=\dfrac{6x^2-11x-2}{3x^2-5x-2}=\dfrac{(6x+1)(x-2)}{(3x+1)(x-2)}$

asymp ↑ ↑ hole at $x=2$

HA: $y=2$
VA: $x=-1/3$

b) $f(x)=\dfrac{1}{x^2+1}$

$x^2+1=0$
no solns, no VA

HA: $y=0$
VA: none

c) $f(x)=\dfrac{4}{x-6}+4$

$$=\frac{4+4x-24}{x-6}$$

HA: $y=4$
VA: $x=6$

3. Suppose that at price p, demand for a certain product is given by $q(p) = \dfrac{(p-100)^2}{2}$.

a) Find the price elasticity of demand when price is \$20. Is demand elastic or inelastic at this price?

$$E(p) = \frac{p}{q} \cdot q' = \frac{p}{\frac{(p-100)^2}{2}} \cdot (p-100) = \frac{2p}{p-100}$$

$$E(20) = \frac{40}{-80} = \frac{-1}{2}$$

$|E(20)| = \frac{1}{2} < 1$, demand is <u>inelastic</u> when $p = 20$.

b) Give an example of a product in the correct price range that might behave as described in (a).

necessity for \$20, lots to choose from...

c) If the price of \$20 decreases by 10%, describe how demand will change.

demand will go <u>up</u> by 5%. [Price will go from \$20 to \$18, and demand will go from 3200 to 3360]

must have this

nice, but not necessary

4. Determine where the function $f(x) = x^3 - 9x^2 + 24x - 19$ is increasing and where it is decreasing, and where it is concave up and concave down. Find all extrema and inflection points. Then sketch the graph.

$$f'(x) = 3x^2 - 18x + 24$$
$$= 3(x^2 - 6x + 8)$$
$$= 3(x-2)(x-4)$$

<u>CN</u>: $x = 2, 4$

$+ \quad - \quad +$ f'

$2 \quad 4$

$$f''(x) = 6x - 18$$
$$= 6(x-3)$$

$- \quad +$ f''

3

inc down | dec down | dec up | inc up — combo

$2 \quad 3 \quad 4$

inc on $(-\infty, 2) \cup (4, \infty)$

dec on $(2, 4)$

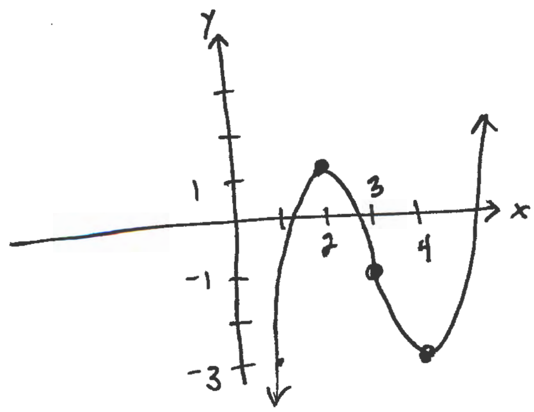
max $(2, 1)$

min $(4, -3)$

conc up on $(3, \infty)$

conc down on $(-\infty, 3)$

inf. pt $(3, -1)$



5. Find all absolute extrema of $f(x)=\dfrac{x}{x^2+1}$ on the interval $[0,2]$.

$$f'(x)=\frac{(1)(x^2+1)-(x)(2x)}{(x^2+1)^2}=\frac{x^2+1-2x^2}{(x^2+1)^2}=\frac{1-x^2}{(x^2+1)^2}$$

CN : $x=1$, $\cancel{x=-1}$ not in interval

$f(0)=0$

$f(1)=1/2$ absolute max $(1, 1/2)$

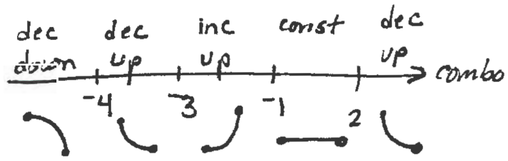
$f(2)=2/5$ absolute min $(0,0)$

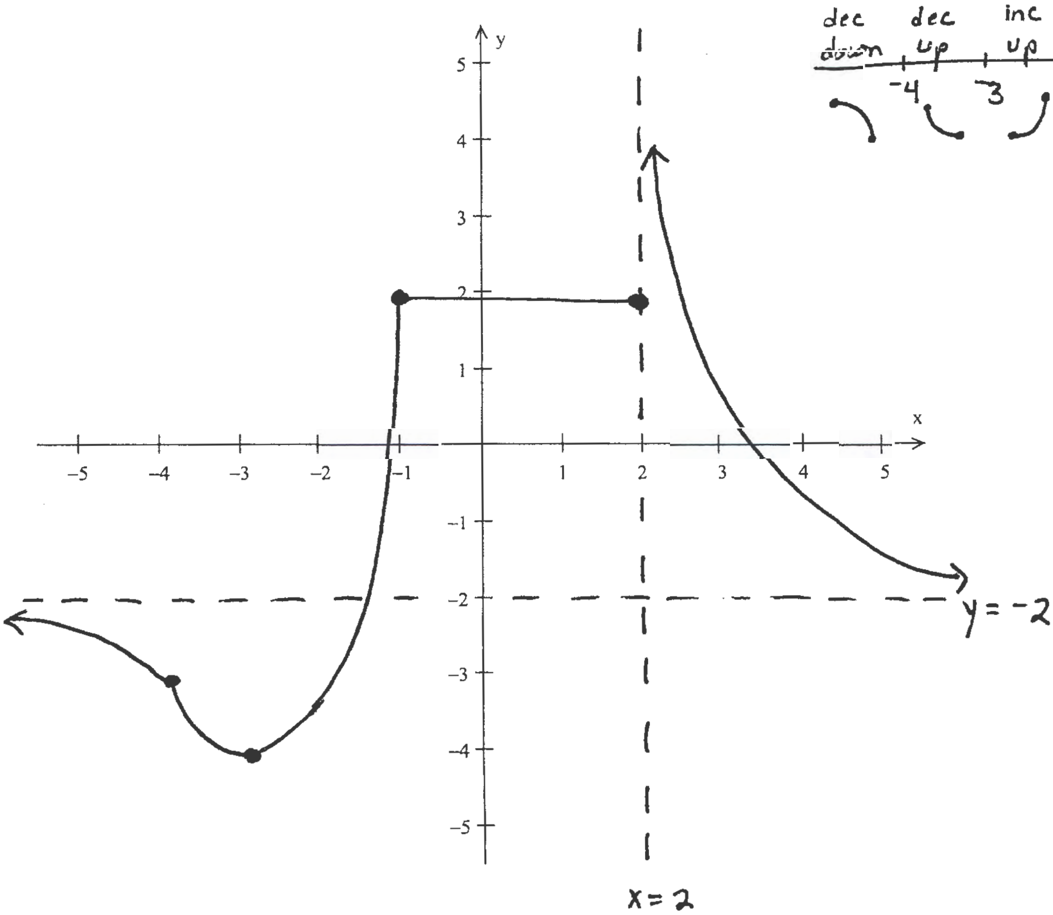
6. Sketch the graph of a function $f(x)$ so that all conditions below are satisfied. Be sure your graph is big enough so I can see it and it is properly labeled.

a) $\lim_{x\to-\infty} f(x)=-2$, $\lim_{x\to 2^+} f(x)=\infty$, and $f(x)$ is defined for all x. HA $y=-2$, VA $x=2$ (on right)

b) $f'(x)<0$ when $x<-3$ and when $x>2$, $f'(x)=0$ when $-1<x<2$, and $f'(x)>0$ when $-3<x<-1$.

c) $f''(x)<0$ when $x<-4$, but $f''(x)>0$ when $-4<x<-1$ and when $x>2$.





7. In a factory, the output Q is given by the equation $Q = 60K^{\frac{1}{3}}L^{\frac{2}{3}}$ units, where K is the capital investment in thousands of dollars, and L is the size of the labor force in worker hours. If output is kept constant, at what rate is capital investment changing at a time when $K = 8$, $L = 1000$, and L is increasing at the rate of 25 worker hours per week?

$$\frac{dQ}{dt} = 60\left[\frac{1}{3}K^{-2/3}L^{2/3}\frac{dK}{dt} + \frac{2}{3}K^{1/3}L^{-1/3}\frac{dL}{dt}\right]$$

$$\frac{dQ}{dt} = \frac{20L^{2/3}}{K^{2/3}}\frac{dK}{dt} + \frac{40K^{1/3}}{L^{1/3}}\frac{dL}{dt}$$

$$0 = 20\frac{(1000)^{2/3}}{8^{2/3}}\frac{dK}{dt} + 40\frac{(8)^{1/3}}{(1000)^{1/3}}(25)$$

$$0 = 20\frac{(100)}{4}\cdot\frac{dK}{dt} + 40\frac{(2)}{10}(25)$$

$$0 = 500\frac{dK}{dt} + 200$$

$$\frac{dK}{dt} = -2/5$$

To keep output the same, capital investment should be changing at a rate of $-2/5$ thousand dollars per week (or \$400 per week decrease)

8. Mrs. Jones runs a small insurance company that sells policies for a large firm. Mrs. Jones does not sell policies herself, but she is paid a commission of \$50 for each policy sold by her employees. When she employs m salespeople, her company will sell q policies each week, where $q = m^3 - 12m^2 + 60m$. She pays her employees \$750 per week, and her weekly fixed costs are \$2500. Her office can accommodate at most 7 employees. How many employees should she have in order to maximize her weekly profit?

Profit = Rev − Cost

$P = 50(m^3 - 12m^2 + 60m) - 750m - 2500$

$P = 50m^3 - 600m^2 + 3000m - 750m - 2500$

$P = 50m^3 - 600m^2 + 2250m - 2500$

$P' = 150m^2 - 1200m + 2250$

$P' = 150(m^2 - 8m + 15)$

$P' = 150(m-3)(m-5)$

CN : $m = 3, 5$

P': $+$ | 3 | $-$ | 5 | $+$

local max when $m=3$

Could have max at endpt.

check end pts:

$P(0) = -2500$

$P(3) = 200$

$P(7) = 1000 \leftarrow$ max

To maximize profit, she should have 7 employees.