NAME KEY

Math 12
Test 2
Summer 2011

You have 60 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Suppose $f(x)=\frac{x^2}{1-x}$. List the intervals where the function is increasing and where it is decreasing, and find all of the maximum and minimum *points*.

$$f'(x) = \frac{2x(1-x)-(x^2)(-1)}{(1-x)^2} = \frac{2x-2x^2+x^2}{(1-x)^2} = \frac{2x-x^2}{(1-x)^2} = \frac{x(2-x)}{(1-x)^2}$$

CN: $x = 0, 1, 2$

$\ominus$ $\oplus$ $\oplus$ $\ominus$ f'

$f(0)=0$

$f(2) = 4/-1 = -4$

inc on $(0,1)\cup(1,2)$

dec on $(-\infty,0)\cup(2,\infty)$

max $(2,-4)$

min $(0,0)$

2. For the following functions, find all horizontal and vertical asymptotes (remember that an asymptote is a LINE, not a number). If there are no asymptotes, say so.

(a) $f(x)=\sqrt{\frac{x^3+1}{4x^3}}$

VA: $4x^3=0$, $x=0$

HA: $\frac{\sqrt{x^3}}{\sqrt{4x^3}} \to \frac{\sqrt{1}}{\sqrt{4}} \to \frac{1}{2}$

VA: $x=0$

HA: $y=1/2$

(b) $f(x)=\frac{3x+3}{x^2-x-2} = \frac{3(x+1)}{(x-2)(x+1)}$

VA: $x=2$ (notice $x=-1$ gives a hole)

HA: $y=0$

(c) $f(x)=\frac{4x-5}{2}$

VA: none

HA: none

3. Suppose $q(p)=\sqrt{2500-2p}$ units of a product are demanded when price is p dollars per unit.

a) Calculate the price elasticity of demand when $p=900$. At this price, is the demand elastic or inelastic?

$$E(p) = q' \cdot \frac{p}{q} = \frac{1}{2}(2500-2p)^{-1/2}(-2) \cdot \frac{p}{\sqrt{2500-2p}}$$

$$E(900) = \frac{1}{2}(2500-1800)^{-1/2}(-2) \cdot \frac{900}{\sqrt{2500-1800}}$$

$$= \frac{-900}{700} = \frac{-9}{7} \longleftarrow \text{elasticity when } p=900.$$

Since $|E(900)| = \left|\frac{-9}{7}\right| = \frac{9}{7} > 1$, demand is elastic.

b) Write a sentence explaining the meaning of your answer in (a) in plain language.

If price goes up 1% from \$900 (to \$909), demand will go down $\frac{9}{7}$%.

c) Give an example of a product in the correct price range that might behave this way.

Any luxury item that costs \$900, maybe airline tickets, jewelry ...

4. Find the derivatives of the following functions:

a) $f(x)=\sqrt[3]{(1-3x)^2} = (1-3x)^{2/3}$

$$f'(x) = \frac{2}{3}(1-3x)^{-1/3}(-3).$$

b) $f(x)=\sqrt{\frac{x+2}{3x-1}} = \left(\frac{x+2}{3x-1}\right)^{1/2}$

$$f'(x) = \frac{1}{2}\left(\frac{x+2}{3x-1}\right)^{-1/2}\left(\frac{(1)(3x-1)-(x+2)(3)}{(3x-1)^2}\right)$$

OR $$f'(x) = \frac{\frac{1}{2}(x+2)^{-1/2}(1)(3x-1)^{1/2} - (x+2)^{1/2}\left(\frac{1}{2}\right)(3x-1)^{-1/2}(3)}{3x-1}$$

5. Sketch a nice BIG graph of a function with all the properties listed below. Make sure your graph is clearly labeled.

a) $f'(x) > 0$ when $-1 < x < 1$, but $f'(x) \leq 0$ otherwise

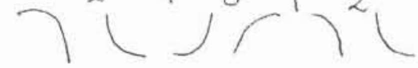
b) $f''(x) > 0$ when $-2 < x < 0$ and when $x > 2$, but $f''(x) \leq 0$ otherwise

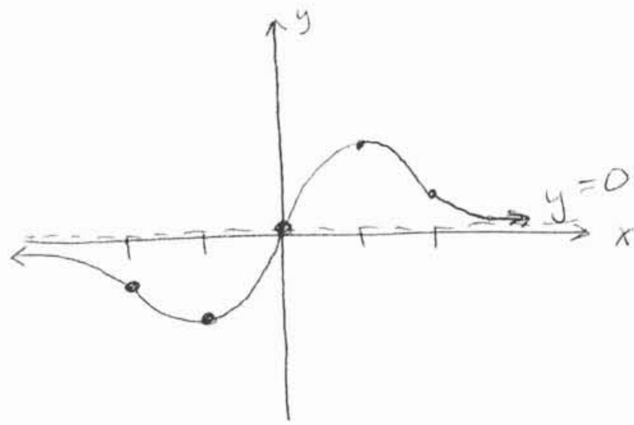
c) $f(x)$ is defined for all values of x

d) $\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = 0$ $\longrightarrow$ HA at $y = 0$, gets close on <u>both</u> ends.

dec down | dec up | inc up | inc down | dec down | dec up $\to$ combo

$-2 \quad -1 \quad 0 \quad 1 \quad 2$





6. Find the equation of the line tangent to the graph of $x^2y - 2xy^3 + 6 = 2x + 2y$ at the point $(0,3)$.

$(2x)(y) + (x^2)(1)(y') + (-2)(y^3) + (-2x)(3y^2)(y') = 2 + 2(1)(y')$

Fill in $(0,3)$, find $y' =$ slope.

$0 + 0 - 2(27) + 0 = 2 + 2y'$

$-54 = 2 + 2y'$

$-56 = 2y'$

$-28 = y' = m$

<u>Line</u>: $y - 3 = -28(x - 0)$

$y = -28x + 3$

7. Find the absolute minimum and absolute maximum *points* of $f(x) = (x^2 - 4)^3$ on the interval $[-3, 1]$.

$f'(x) = 3(x^2-4)^2(2x)$

$= 3[(x+2)(x-2)]^2(2x)$

CN: $x = -2, 2, 0$ not in interval

$f(-3) = (9-4)^3 = 125 \leftarrow$ abs max at $(-3, 125)$

$f(-2) = (4-4)^3 = 0$

$f(0) = (0-4)^3 = -64 \leftarrow$ abs. min at $(0, -64)$

$f(1) = (1-4)^3 = -27$

8. A commercial fruit grower must decide when to pick his pears. If he does it now, the pears will bring 32 cents per pound and each tree will yield 60 pounds of pears. Over the next 4 weeks, the yield per tree will increase 9 pounds per week, but the price per pound will decrease 3 cents per week. In order to maximize revenue, when should he pick to pears? (Hint: It might be easier to work in pennies rather than dollars).

Revenue = price · quantity

$R = (32 - 3x)(60 + 9x)$, where x = # weeks from now.

$R' = (-3)(60+9x) + (32-3x)(9)$

$= -180 - 27x + 288 - 27x$

$= -54x + 108 = 0$

CN: $x = 2$

must make sure $x=2$ gives a maximum

method ①

$+ \quad - \quad R'$

2

max

method ②

$R'' = -54$

$R''(2) = -54 < 0$ max.

Should wait $\boxed{2 \text{ weeks}}$ to pick pears in order to maximize revenue.