NAME Key

Math 12
Test 3
Fall 2012

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Solve $y' = y^2 e^{3x}$ if $y = 1$ when $x = 0$.

$\frac{dy}{dx} = y^2 e^{3x}$

$\int y^{-2}\,dy = \int e^{3x}\,dx$

$-y^{-1} = \frac{1}{3} e^{3x} + C$

$-1 = \frac{1}{3} + C$

$C = -4/3$

$-\frac{1}{y} = \frac{1}{3} e^{3x} - \frac{4}{3}$

$\frac{1}{y} = \frac{4}{3} - \frac{1}{3} e^{3x}$

$y = \frac{1}{\frac{4}{3} - \frac{1}{3} e^{3x}}$

2. Find y':

(a) $y = \sqrt{x \ln(x^2+1)} = (x \ln(x^2+1))^{1/2}$

$y' = \frac{1}{2}(x \ln(x^2+1))^{-1/2}\left(\ln(x^2+1) + x\left(\frac{2x}{x^2+1}\right)\right)$

(b) $y = x^2 e^{3x-2}$

$y' = 2xe^{3x-2} + x^2 e^{3x-2}(3)$

3. Which is the better investment option, (A) an account earning annual interest of 8% compounded quarterly, or (B) an account earning annual interest of 7.5% compounded continuously?

use $P=1$, $t=1$:

(A) $B = P\left(1+\frac{r}{k}\right)^{kt}$

$B = \left(1 + \frac{.08}{4}\right)^4$

$B = (1.02)^4$

$B \approx 1.0824$

Account A is better.

(B) $B = Pe^{rt}$

$B = e^{0.075}$

$B \approx 1.0779$

4. A fossil is found to contain 1/6 of its original ^{14}C. The half-life of ^{14}C is 5730 years. How old is the fossil?

① $t=0$ $\quad B = P$

② $t=5730$ $\quad B = \frac{1}{2}P$

③ $t=?$ $\quad B = \frac{1}{6}P$

① $B = Pe^{rt}$

$P = Pe^{0}$ ... no extra info.

② $\frac{1}{2}P = Pe^{r(5730)}$

$\frac{1}{2} = e^{5730r}$

$\ln\frac{1}{2} = 5730r$

$r \approx \frac{\ln 1/2}{5730}$

$r \approx -0.00012097$

③ $\frac{1}{6}P = Pe^{-0.000121t}$

$\ln 1/6 = -0.000121t$

$t = \frac{\ln 1/6}{-0.000121}$

$t \approx 14{,}812$ years old

5. Solve the following for x:

a) $3^{x^2-4x} = \left(\frac{1}{81}\right)^{x-4}$

$$3^{x^2-4x} = 3^{-4(x-4)}$$
$$x^2-4x = -4x+16$$
$$x^2 = 16$$
$$x = \pm 4$$

b) $log_3(x-2)+log_3(x+6)=2$

$$\log_3 (x-2)(x+6) = 2$$
$$x^2+4x-12 = 3^2 = 9$$
$$x^2+4x-21 = 0$$
$$(x-3)(x+7) = 0$$
$$x = 3, \cancel{x=-7} \text{ not in domain}$$

c) $log_x(2x-3)=1$

$$2x-3 = x^1$$
$$x = 3$$

6. Evaluate the following integrals:

a) $\int \frac{1}{x \ln x} dx$ $= \int \frac{1}{x} \cdot \frac{1}{\ln x} dx = \int \frac{1}{u} du = \ln|u| + C$

$= \ln|\ln x| + C$

Let $u = \ln x$

$du = \frac{1}{x} dx$

b) $\int \frac{2x^2+x-5\sqrt{x}}{3x^2} dx$

$$= \frac{1}{3}\int (2x^2+x-5x^{1/2})(x^{-2})\, dx$$
$$= \frac{1}{3}\int (2+x^{-1}-5x^{-3/2})\, dx$$
$$= \frac{1}{3}\left(2x + \ln|x| - \frac{5x^{-1/2}}{-1/2}\right) + C$$
$$= \frac{2}{3}x + \frac{1}{3}\ln x + \frac{10}{3}x^{-1/2} + C$$

7. For the function $f(x) = xe^x$, list all intervals of increase and decrease, all maximum and minimum *points*, intervals where the function is concave up and concave down, all inflection *points*, and all asymptotes (or say there are none). Then sketch the graph of the function.

$f'(x) = e^x + xe^x = e^x(1+x)$

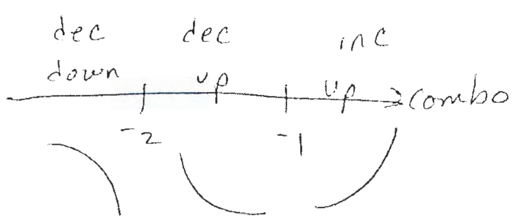
CN : $x = -1$ $\quad \underset{-1}{\overset{-\quad +}{\longrightarrow}} f'$

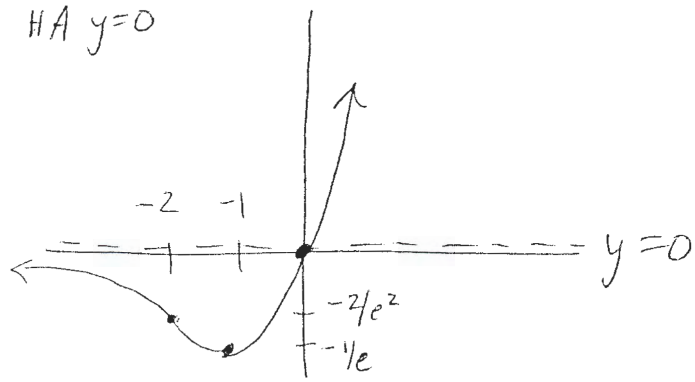
$f''(x) = e^x + e^x + xe^x = e^x(2+x)$

IN : $x = -2$ $\quad \underset{-2}{\overset{-\quad +}{\longrightarrow}} f''$

inc $(-1, \infty)$
dec $(-\infty, 1)$
min $(-1, \frac{-1}{e})$
conc up $(\cancel{-\infty}\ -2, \infty)$
conc down $(-\infty, -2)$
inf pt $(-2, -2/e^2)$
VA : none
HA : $y = 0$

Asymp : defined for all x, no VA
If $x \to \infty$, $y \to \infty$
If $x \to -\infty$, $y \to 0$, HA $y = 0$





8. Evaluate $\int x^3 \ln x \, dx$.

Let $u = \ln x$ $\qquad dv = x^3\,dx$

$du = \frac{1}{x}dx$ $\qquad v = \int x^3 dx = \frac{1}{4}x^4$

$uv - \int v\,du = \frac{1}{4}x^4 \ln x - \int \frac{1}{4}x^4 \cdot \frac{1}{x}\,dx$

$= \frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3 dx$

$= \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$