COMPOSITE GYROMAGNETIC MATERIAL AS A LORENTZIAN MEDIUM FOR FDTD MODELING

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Abstract: This paper presents the effective constitutive parameters of dispersive gyromagnetic composites, particularly mixtures of high-anisotropic hexagonal ferrite particles, in the form suitable for FDTD modeling using recursive convolution procedure. The frequency dependence of both permittivity and permeability are represented in Lorentzian "narrowband" and "wideband" forms, depending on the relation between the resonance frequency and resonance curve width. The procedure for updating electric and magnetic field components using such media is shown.

INTRODUCTION

Composite gyromagnetic media (GM) containing particles with high internal field of magnetic anisotropy (hexagonal ferrites) are applied in microwave absorbers, filters of harmonics, matched loads, isolators, etc. Their frequency characteristic can be modified as desired by varying the contents of the composite, and they do not need an external magnetization field for their operation over wide frequency ranges due to the phenomenon of natural ferromagnetic resonance (NFMR) [1].

The robust finite-difference time-domain (FDTD) technique can be rather efficient for calculating fields in structures containing GM. The effective electrodynamic parameters of composite media containing particles with high internal field of magnetic anisotropy (hexagonal ferrites) can be represented in the form convenient for FDTD analysis using recursive convolution approach introduced by Luebbers [2].

RECURSIVE CONVOLUTION IN FDTD ANALYSIS FOR GM

Hexagonal ferrite mixture constitutive parameters are represented in Lorentzian form. It can be a one-pole (one-resonance) curve if the material is a single-component (with one type of a hexagonal ferrite filler), or multi-pole (with several resonance peaks) if the material is multi-component (contains mixture of ferrites of different types). For a gyromagnetic composite media the magnetic susceptibility in frequency domain can be represented as [1],

$$c_m(w) = \frac{2}{3}\sum_{i=1}^{N_F} \frac{f_i w_{Mi} \overline{w}_{Ai}}{\overline{w}_{Ai}^2 + 2jww_{Li} - w^2}, \qquad (1)$$

where $w_{Mi} = gm_0 M_{Si}; \overline{w}_{Ai} = gm_0 \overline{H}_{Ai}; w_{Li} = gm_0 DH_i/2$; f_i is volumetric fraction of ferrite powder of type i, M_{Si} is saturation magnetization, $g = 1.76\cdot 10^{11}$ C/kg; $\overline{H}_{Ai}$ is an average value internal field of crystallographic anisotropy for a ferrite powder of type i, and w_{Li} is the loss parameter. $\overline{w}_{Ai}$ and w_{Li} are found using methods of mathematical statistics if the corresponding probability density function (for example, Cauchy distribution) is known [3]. Every Lorentzian peak can be considered as either "narrowband" or "wideband", depending on the relation between the resonance frequency and width of the corresponding resonance curve. It will be shown below that recursive convolution algorithms in these two cases differ.

"Narrowband" Lorentzian GM. If the material is "narrowband" ($\overline{w}_{Ai} > w_{Li}$), then time-domain susceptibility kernel found as a Fourier (or Laplace) transform using Heaviside's formula is

$$c_m(t) = \sum_{i=1}^{N_F} G_i \exp(-w_{Li}t)\sin(w_i t)u(t), \qquad (2)$$

where $G_i = \frac{2}{3} \cdot \frac{f_i w_{Mi} \overline{w}_{Ai}}{w_i}$ and $w_i = \sqrt{\overline{w}_A^2 - w_{Li}^2}$. Susceptibility kernel in complex exponential form is

$$c_m(t) = \sum_{i=1}^{N_F} \mathrm{Re}\{G_i \exp(\tilde{g}_i t)\}, \qquad (3)$$

where $\tilde{g}_i = -w_{Li} + jw_i$. According to the FDTD algorithm H-filed is updated as [5]

$$\vec{H}^{n+1}(m) = A_m \vec{H}^n(m) - B_m \nabla_d \times \vec{E}^n[m, m+1] - \mu_0 \sum_{i=1}^{N_F} \mathrm{Re}\left(\tilde{\vec{F}}_i^n(m)\right), \qquad (4)$$

where coefficients are $A_m = \frac{2 - \sum_{i=1}^{N_F} c_{mi}^0 \Delta t}{2 + \sum_{i=1}^{N_F} c_{mi}^0 \Delta t}$; $B_m = \frac{2\Delta t}{2 + \sum_{i=1}^{N_F} c_{mi}^0 \Delta t}$; $(\nabla_d \times)$ is central-difference discrete curl operator, and "static" susceptibility parameter is

$$c_{mi}^0 = \mathrm{Re}\int_0^{Dt} \tilde{c}_{mi}(t)dt = \frac{G_i}{|\tilde{g}|^2}\left(w_{Ai}\left(1 - \exp(-w_{Li}D t)\right)\cos\overline{w}_{Ai}D t - w_{Li}(\exp(-w_{Li}D t)\sin\overline{w}_{Ai}D t\right) \qquad (5)$$

Complex discrete convolution function in (4) is

$$\tilde{\vec{\Phi}}_i^n(m) = \sum_{k=0}^{n-1} \vec{H}^{n-k}(m)\Delta\tilde{c}_{mi}^{k+1}, \qquad (6)$$

where $\Delta\tilde{c}_{mi}^{k+1} = \tilde{c}_{mi}^{k+1} - \tilde{c}_{mi}^{k+2} = \int_{(k+1)\Delta t}^{(k+2)\Delta t} \tilde{c}_{mi}(t)dt - \int_{(k+2)\Delta t}^{(k+3)\Delta t} \tilde{c}_{mi}(t)dt$. (7)

Because of the exponential function (3), the value $\Delta\tilde{c}_{mi}^{k+1}$ is calculated recursively,

$$\Delta\tilde{c}_{mi}^{k+1} = \exp(\tilde{g}_i k\Delta t)\Delta\tilde{c}_{mi}^k . \qquad (8)$$

Convolution function is calculated recursively, too,

$$\tilde{\Phi}_i^{n+1}(m) = E^{n+1}(m)\Delta\tilde{c}_i^0 + \exp(\tilde{g} \cdot \Delta t) \cdot \tilde{\Phi}_i^n(m), \qquad (9)$$

where $\Delta\tilde{c}_{mi}^0 = -\frac{G_i}{\tilde{g}_i}\left(1 - \exp(\tilde{g}_i k\Delta t)\right)^2$.

If real and imaginary parts in (9) are separated, two related recursive equations are obtained

$$Re\left(\tilde{\vec{F}}_i^{n+1}(m)\right) = \vec{H}^{n+1} \cdot Re\left(\Delta\tilde{c}_i^0\right) + e^{-w_{Li}\Delta t}\left[Re\left(\tilde{\vec{F}}_i^n(m)\right) \cdot \cos w_i\Delta t - Im\left(\tilde{\vec{F}}_i^n(m)\right) \cdot \sin w_i\Delta t\right],$$

$$Im\left(\tilde{\vec{F}}_i^n(m)\right) = \vec{H}^n \cdot Im\left(\Delta\tilde{c}_i^0\right) + e^{-w_{Li}\Delta t}\left[Im\left(\tilde{\vec{F}}_i^{n-1}(m)\right) \cdot \cos w_i\Delta t + Re\left(\tilde{\vec{F}}_i^{n-1}(m)\right) \cdot \sin w_i\Delta t\right] \qquad (10)$$

To calculate function $\tilde{\Phi}_i^{n+1}(m)$ saving real and imaginary parts of the function at time steps n and $(n-1)$ is needed.

"Wideband" Lorentzian GM. When $\overline{w}_{Ai} \le w_{Li}$, the susceptibility kernel is non-oscillating damping function

$$c_m(t) = \sum_{i=1}^{N_F} G_i^W \cdot \exp(-w_{Li}t)\sinh(w_i^W \cdot t)u(t), \qquad (11)$$

where $w_i^W = \sqrt{w_{Li}^2 - \overline{w}_A^2}$, $G_i^W = \frac{2}{3} \cdot \frac{f_i w_{Mi} \overline{w}_{Ai}}{w_i^W}$.

"Static" susceptibility is calculated as in (5) and is a real value

$$c_{mi}^0 = \frac{G_i^W}{2} \left(\frac{e^{(w_i^W - w_{Li})Dt} - 1}{w_i^W - w_{Li}} + \frac{e^{-(w_i^W + w_{Li})Dt} - 1}{w_i^W + w_{Li}} \right), \tag{12}$$

Convolution function (9) can be represented in the form of two separate recursively calculated terms, $\vec{\Phi}_i^n = \vec{\Phi}_{i(1)}^n + \vec{\Phi}_{i(2)}^n$, where

$$\vec{F}_{i(1)}^n = \vec{H}^n Dc_{mi(1)}^0 + e^{(w_i^W - w_{Li})D t} \cdot \vec{F}_{i(1)}^{n-1}, \tag{13}$$

$$\vec{F}_{i(2)}^n = \vec{H}^n Dc_{mi(2)}^0 + e^{-(w_i^W + w_{Li})Dt} \cdot \vec{F}_{i(2)}^{n-1}. \tag{14}$$

and

$$Dc_{mi(1,2)}^0 = -\frac{G_i^W}{2} \left(\frac{\left(e^{-(w_i^W \mp w_{Li})D t} - 1 \right)^2}{w_i^W \mp w_{Li}} \right). \tag{15}$$

For recursive convolution function updating with "wideband" material it is necessary to save pair of values $\vec{F}_{i(1,2)}$ at previous time step.

CONCLUDING REMARKS

FDTD modeling of isotropic composite gyromagnetic medium can use recursive convolution procedure, if magnetic susceptibility kernel of the medium is represented as a sum of complex exponents of time. Multi-pole Lorentzian model is used for the representation of magnetic susceptibility of the gyromagnetic composite multi-component media. Two types of Lorentzian models are considered – "narrowband" and "wideband" – dependent on the ratio of the material bandwidth to the resonance frequency. Formulas for recursive convolution used in FDTD algorithm fields updating are obtained. Compared to Debye material model, both "narrowband" and "wideband" Lorentzian models of material parameters representation demand more memory, but more general and allow taking into account resonance effects in materials of different nature. If the material exhibits frequency dependence of dielectric properties in the same frequency range as magnetic properties, then combined algorithm for both electric field and magnetic field updating using recursive convolution technique should be used.

REFERENCES

[1] M.Koledintseva, L.Mikhailovsky, A.Kitaytsev. "Advances of Gyromagnetic Electronics for EMC problems". IEEE Symp. On EMC, Washington, DC, Aug.21-25, 2000. V. 2, p. 773-778.

[2] K.Kunz, R.Luebbers. *"The Finite Difference Time Domain Method for Electromagnetics"*. CRC Press, Inc., 1993.

[3] A.A Kitaytsev, M.Y.Koledintseva, V.P.Cheparin, A.A Shinkov. Electrodynamic parameters of composite gyromagnetic material based on hexagonal ferrites. Proc.URSI Symposium on Electromagnetic Theory EMT'98, Greece, Thessaloniki, V.2, p.790-793.